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Effects of Divergence Shape on the Characteristics of Hydraulic Jump in Stilling Basins Using Numerical Simulation and Neural Networks

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ABSTRACT

Measures such as sudden cross-sectional divergence is among the factors affecting the characteristics of hydraulic jump. If for any reason it is not possible or cost-effective to provide the depth required for the hydraulic jump, then gradual or sudden flow cross-sectional divergence can be a good way to reduce the depth required for the jump. In this research, using the neural network and FLOW-3D numerical model, a three-dimensional (3D) model for fluid simulation, the effect of sudden divergence stilling basin on characteristics was simulated. The results of the neural network are very close to the physical model. The study revealed that 3D simulation using Flow-3D software could simulate a hydraulic jump with an average error of 2.41%. The efficiency of stilling basins divergent was calculated to be 71%, which is higher than the classic stilling basins with an efficiency of 53.3%. Depth after jumping in divergent stilling basins modeled at 27.8 and 41.4 l/s was found to be 12 and 25% less than classic basins, respectively. Compared to the classical mode in the divergent stilling basins parabolic, gradual, and sudden, the decrease in jump length was found to be 25.9%, 27.5%, and 31.8%, respectively. The results showed that the sudden divergent stilling basin has the best performance in terms of hydraulic parameters.

Keywords: Neural Network, Simulation, Stilling Basin, Flow-3D, Divergence

INTRODUCTION

The passage of water over the overflow, below the valve, as well as the change of waterway slope from a steep slope to a gentle slope over a short length, will lead to the formation of a hydraulic jump. A hydraulic jump is a change of the supercritical state of the flow to the subcritical state, associated with a great loss of kinetic energy. In this flow, the shallow depth of the current is converted to the downstream depth at short distances, which is sub-critical depth, and the downstream energy decreases downstream (Izadjoo and Shafaei Bajestan, 2004). Increasing the depth of flow in a short distance and decreasing the flow velocity downstream is accompanied by high turbulence and turbulence, which gradually decreases downstream of this turbulence and turbulence of the water. As a result, the turbulence of the airflow enters the water. As the secondary jump depth approaches the downstream depth, the air bubbles transferred to the downstream will disappear. To control the hydraulic jump, kinetic energy-consuming structures are used, the most common of which is stilling basin (Bakhtiari and Kashefipour, 2005). Stilling basin is a part of a flooring channel built downstream of dams or valves, and its purpose is to form a jump inside the basin. Optimal stilling basin design and reducing the operating cost require reviewing the jump characteristics and determining the effective jump parameters involved in the design of this structure. Measures such as sudden cross-sectional divergence are among the factors affecting hydraulic jump characteristics (Bakhtiari and Kashefipour, 2007). Optimal and effective performance of classic stilling basin requires proper water depth supply downstream. If for any reason (economic cost or drilling problems at the stilling basin), the depth required for hydraulic jumping is not possible or cost-effective, gradual or sudden flow cross-sectional divergence can be a good way to reduce the depth required for the jump. At the same time, it reduces the cost of building a stilling basin. The first series of jump experiments in sudden cross-sectional divergence was performed by Shojaeian (2010). Rajaratnam provided general equations for the sequential depth ratio that did not agree well with Blanger's general equation for the classical jump (Rajaratnam, 1967). Rajaratnam studied R-jump in sudden cross-sectional divergence in Froude numbers between 2 and 9 with an expansion ratio between 0.3 and 0.9. They concluded that the sequential depth in the initial Froude equation escape jump is the expansion ratio (Rajaratnam, 1968; Rajaratnam and Subramanya, 1968).

In T-jump laboratory studies, Hager (1992) performed experiments in two channels, the first 0.5 m wide and the second 1.5 m wide 0.7 m high and 11 m long. They used 0.9 m long plates in the first channel to create divergence with expansion ratios of 0.2, 0.33, 0.5 and 0.67 and in the second channel for expansion ratios of 0.33 and 0.5. In this study, the channel divergence was symmetric only for the 0.33 expansion ratio. In the other expansion ratios, the divergence channel was asymmetric. This study was also conducted in the range of Initial Froude numbers 2.5 to 10 with an initial depth between 0.13 to 0.6 m. The study showed that the value of sequential depth ratio depends on the initial Froude number, expansion ratio and toe location (Hager, 1992; 1989; 1993).

Abdulmatin et al. (2008) presented the equation for the sequent depth ratio. As mentioned earlier, the modified Froude number depends on two parameters, k1 and k2. The coefficient k1 is a coefficient that depends on the location of the jump, the sequential depth ratio and the expansion ratio. But the coefficient k2 depends only on the sequential depth ratio and expansion ratio. Therefore, for S-jump, since the place of formation of the jump is at the point of change of section, i.e., e = 1, then k1 = 0 and the modified Froude number will only be a function of the coefficient k2. Ferreri and Nasselo examined the hydraulic jump in sudden cross-sectional divergence by considering thresholds for three expansion ratios (0.33, 0.5, and 0.25), five Initial Froude numbers, and five different initial depths. They concluded that the jump characteristics depend on s, s / y1, Fr1 and B, where s is the height of the threshold (Ferreri and Nasello, 2002). In an S-jump study, Al-Hamid examined the jump characteristics in Horizontal and sloping channels with sudden divergence with three expansion ratios of 0.33, 0.5 and 0.67 and three-floor slopes of 0.4, 0.25 and zero, in the range of Froude numbers 2.7 to 7.5. He performed his experiments on a flume 10.5 meters long, 30 meters wide and 45 cm deep. He showed that with increasing divergence ratio, the sequential depth ratio increases for different Fr1. However, under all conditions, it is less than the corresponding value in the classic jump (Alhamid, 1994; Alhamid, 2004).

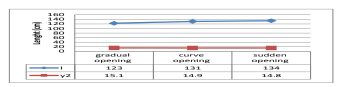
Rezaul Hassan and Abdul Matin (2009) then performed experiments at the Hydraulic and River Engineering Laboratory of the Department of Water Resources Engineering at Dhaka University in Bangladesh. They used a horizontal flume of 12.2 m in length and 0.3 m in height with three expansion ratios of 0.5, 0.67 and 0.38 with an Initial Froude number of 1.33-3.48, an aperture of 3.6-7.3 cm, and a flow rate of 5-19.6 l / s. They concluded that the parameter K2 is dependent on the two parameters B and Fr1 and decreases with increasing expansion ratio. Also, for an expansion ratio greater than 0.67, this parameter will be independent of the Initial Froude number Rezaul Hassan and Abdul Matin (2009).

This study was aimed to simulate the use of the neural network and FLOW-3D numerical model, a three-dimensional (3D) model for fluid simulation, the effect of sudden divergence stilling basin on characteristics.

MATERIAL AND METHODS

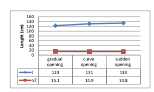
Dimensional analysis

Hager and Bremen (1993) by studying the T-jump, at expansion ratios of 0.2, 0.33, 0.5, and 0.67 and initial Froude numbers 2.5 to 10, measured the jump length equal to x1 + xj and the following equation Presented by:



(1)

The above equation for S-jump, whose jump length contains only xj, will be as follows:



(2)

For T-jump, the jump length is always longer than the classic jump length. This difference will be more significant with decreasing expansion ratio and proximity of the jump toe to the cross-section change, i.e., the tendency of the jump from type T to type S (Hager and Bremen, 1993).

According to previous studies in the field of the jump, sudden divergent stilling basin has the greatest effect on energy dissipation. To investigate the effect of divergence shape on hydraulic jump characteristics, this phenomenon was simulated using neural network and Flow-3D software. It was then calibrated and compared with the physical model located in the hydraulic laboratory of the Faculty of Water Engineering, Shahid Chamran University of Ahvaz. Figure 1 shows the laboratory model image. This model includes the main part of the flume with a length of 12 meters, width of 80 cm and height of 70 cm, sliding valve at the inlet section, downstream sliding valve that regulates the jump toe, pressure supply tank, flow stiller, water outlet guide channel and water supply tank (Kobra Nisi and Dezfuli, 2014).

Figure 1. Laboratory model image.

To create a sudden divergence in the flume cross-section, the channel can be divergent symmetrically (narrowing on both sides of the channel) or asymmetrically (narrowing on one side of the channel). Therefore, the experiments were performed on a flume 12 cm long, 0.8 m wide, and 0.7 m high made of glass and Plexiglas. The sliding valve was used to form a jump and create a supercritical flow. To prevent the contraction of the outlet flow lines from the valve and also that the initial depth of the jump is equal to the expansion of the valve, the upstream shape of the valve has been semicircular. Another sliding valve was used downstream of the flume to stabilize the jump position (Nisi and Shafaei Bajestan, 2013).

An introduction to Flow-3D software

Flow-3D software is multifaceted software that adapts to complex flow conditions in 2D and 3D modeling. This software is dedicated to computational fluid dynamics (CFD) and is provided by Flow Science. The method of solving equations in this software is based on the finite volume method. Mathematical models are one of the most powerful tools in solving complex equations related to fluid mechanics. Today, with the increasing speed of computers, the use of these models has expanded significantly. One of the advantages of mathematical models over physical models is that they are less expensive. In comparison, various changes, such as structural geometry changes, are easily possible in these models. The use of such software due to the high graphic ability and providing three-dimensional flow results according to any geometric shape of the hydraulic structure can provide much information to the designer to optimize the structure's geometric shape. It also causes fewer changes in the hydraulic model and study time, or even in some cases, replaces the construction and conduct of hydraulic studies (Nisi and Shafaei Bajestan, 2013).

To model the desired flume, a 3D view of the flume and the catchment was designed in AutoCAD software and saved in st l format, as shown in Figure 2.

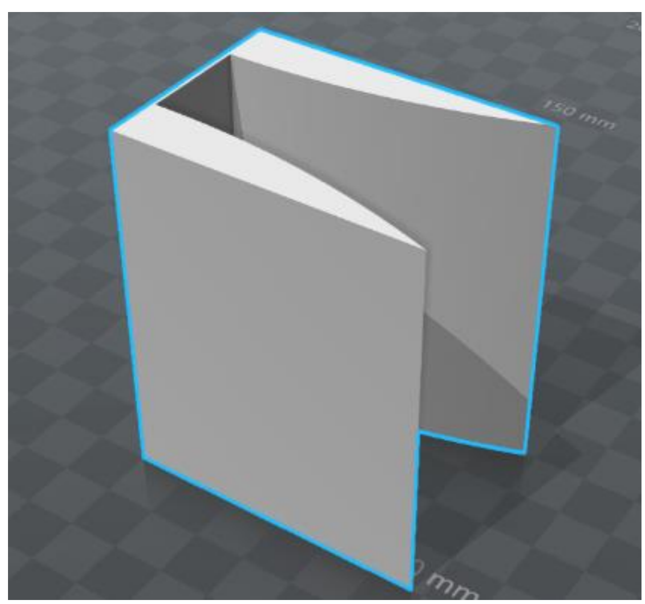


Figure 2. A 3D model designed by AutoCAD software.

A large number of mesh cells were used to model the model. Since increasing the number of mesh cells directly affects the time of each software run, we tried to achieve a certain limit on the size and number of mesh cells to be considered both accurately and simultaneously. These mesh divisions do not affect flow lines and are used only for meshing. The meshes will also vary in different directions of the coordinate axis based on the blocks' length. After determining the number of meshes, Flow-3D software allows the user to determine the correctness of the proportion of their number in all three directions X, Y, Z by two parameters maximum adjacent cell size ratio and maximum Aspect Ratios in different directions. These two parameters are located in the Info section of the Meshing section. The Flow 3D model is shown in Figure 3.

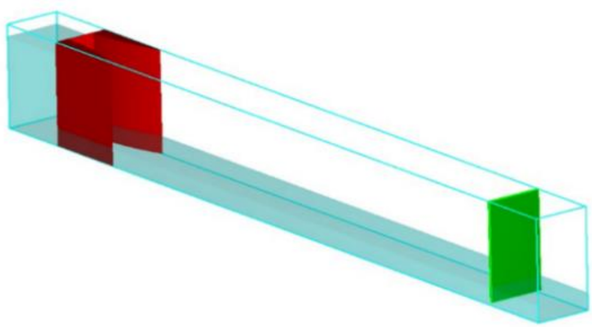


Figure 3. Flow -3D model.

In this research, after performing initial configurations and networking and determining the boundary conditions, the software was run. After calibrating the software, we performed the next runs. After comparison, the boundary conditions were determined as follows: Input border: Volume Flow Rate, Output border: Out Flow; Side borders: wall; Floor border: wall; Upper limit: Specified Pressure using the "use fluid fraction" option.

The simulation with the Flow-3D mathematical model includes two discharges of 27.4 and 41.4 l/s and three types of curved divergence, gradual and divergence. A total of 6 scenarios are shown in Table 1.

Table 1. Flow-3D simulation variables.

Total	**Divergence (3 variables)**	**Flow rate (2 variables)**
Six scenarios	Curved, gradual and divergent	27.4 and 41.4 l/s

Model calibration

To calibrate the model, experiments were performed using observational data in different modes of roughness coefficient and turbulence models. The details of the tests are as follows:

Manning roughness coefficient (n) values: 0.02, 0.025, 0.03, 0.035 and 0.04

Important and common turbulence models used in this software :(Laminar, Prandtl Mixing Length Model, K-Epsilon Model) and inlet flow: 27.4 l/s

Calibration experiments were performed on curved divergence. Each experiment was performed with initial data. In each experiment, water height and jump length in the software were calculated from the obtained results. After importing the results into Excel software, they were compared with the observed data. In each experiment, the amount of error was calculated. According to the calibration experiments results, the lowest error in the experiments was obtained using the Manning roughness coefficient 0.035 and Prandtl Mixing Length Mode model. The error rate, in this case, is -1.45%. Now, using the values obtained in the calibration process, the main tests were performed.

Artificial neural network (ANN)

During the last decade, with the advent of intelligent processing systems such as ANNs, genetic algorithm (GA), fuzzy logic, chaos theory, etc., the discussion of black-box models has entered a new field. The high capability of these models in understanding the intrinsic relationships between data, each inspired by nature and biological systems, is the reason for the high volume of current relevant research around the world (ASCE II, 2000). Each of these models alone has shown its ability to simulate complex processes. There are many areas of research for them. Nevertheless, combining these types of intelligent processing systems and presenting a hybrid model is one of the completely new issues at the international level and can summarize the positive features of these methods in one model. The extraordinary power of ANNs in learning and generalizing and establishing a nonlinear relationship between input and output spaces, as well as its extensive and distributed structure, has made this model always a key element in intelligent hybrid models. On the other hand, GA's potential ability to solve optimization problems creates the belief that a very robust information processing model can identify parameters affecting the phenomenon and eliminate ineffective parameters can be achieved by combining it with ANN. Such a model is called the artificial neural network (GA) model or the evolutionary neural network model. Figure 4 shows the functional simulation of artificial and natural neural networks.

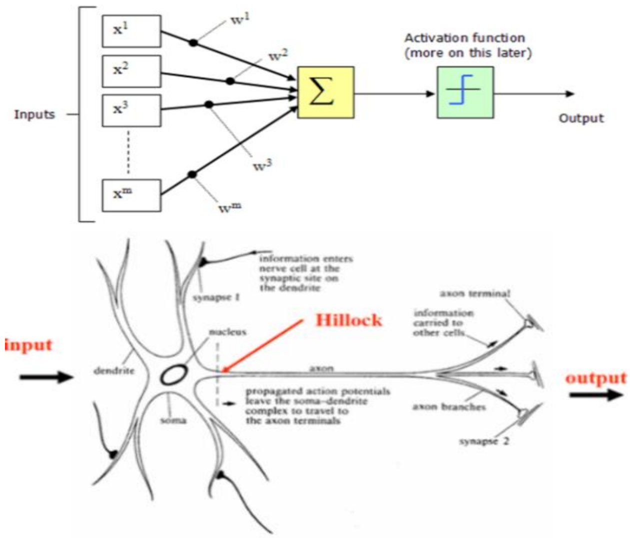


Figure 4. Artificial and natural neural network.

In this research, after entering the information in the neural network and training the program, the information was processed, the results of which will be discussed in the results section. The type of input and output information is shown in Figure 5.

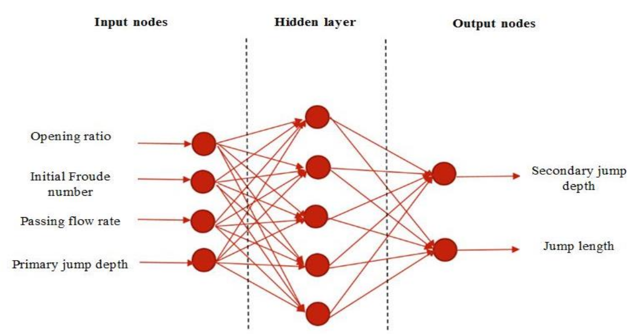


Figure 5. Artificial neural network model used in this study.

MATERIAL AND METHOD

To investigate the effect of divergence shape on hydraulic jump characteristics, this phenomenon was simulated using neural network and Flow-3D software. It was calibrated and compared with the physical model. Figure 6 shows the research method.

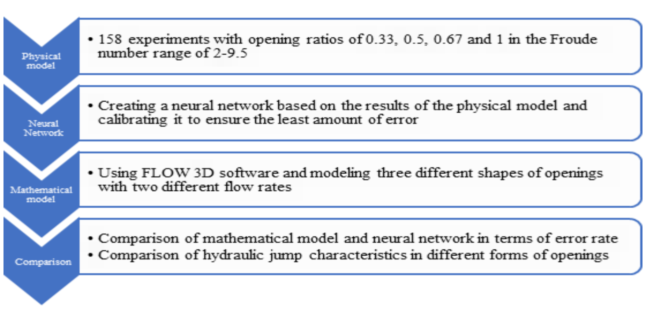


Figure 6. Research methodology.

RESULTS

The obtained results include the results of the flow in two forms: graphic and table. The graphical results include the basin view and how the desired hydraulic parameters, such as velocity, are distributed in the computational model.

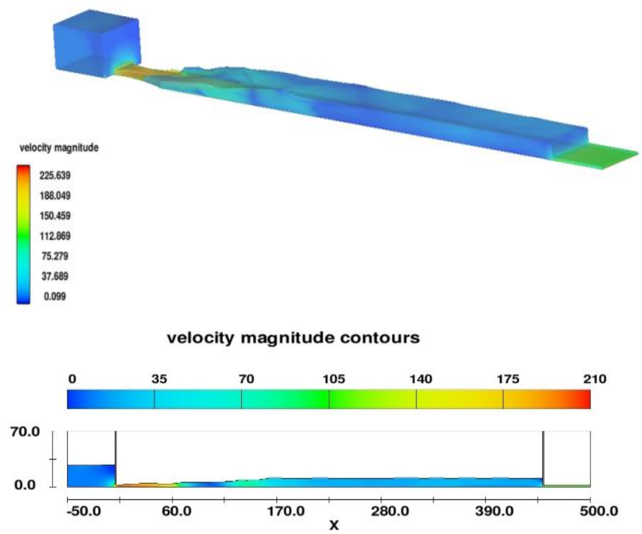


Figure 7. Graphical results of velocity in curved divergence.

As can be seen from Figure 7, the results of the speed in the hydraulic jump can be seen, as well as the length of the jump. The 3D numerical simulation outputs performed for different expansions are in Table 2.

According to the diagram in Figures 8 and 9, it can be seen that the expansion of the secondary jump depth from 27.8 l/s increased from 14.8 to 15.1 cm and from 41.4 l/s from 18.9 to 19.4 cm by changing the expansion from gradual to curved. The hydraulic jump length in Flow rate decreased from 27.8 l/s from 134 to 123 cm and in Flow rate 41.4 l/s from 189 to 174 cm. The change of expansion from gradual to sudden curved of the secondary jump depth has increased the energy drop, which has increased the secondary jump depth and decreased the hydraulic jump length.

Table 2. 3D simulation results.

Type	Flow rate (L/s)	Post-jump depth (cm)	Jump length (cm)
Curved opening	27.8	14.9	131
	41.4	19.1	185
Gradual opening	27.8	14.8	134
	41.4	18.9	189
Sudden opening	27.8	15.1	123
	41.4	19.4	174

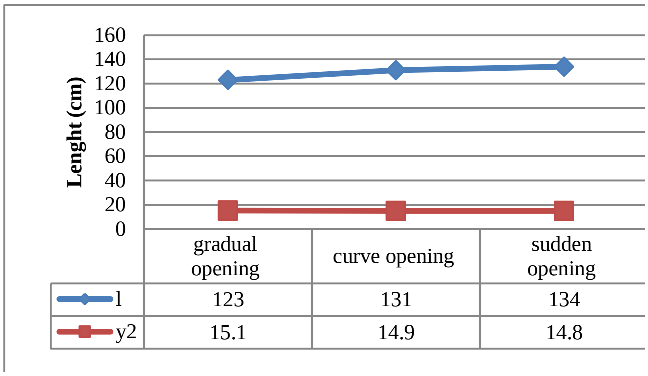


Figure 8. Comparison diagram of secondary jump length and depth at a flow rate of 27.4 l/s.

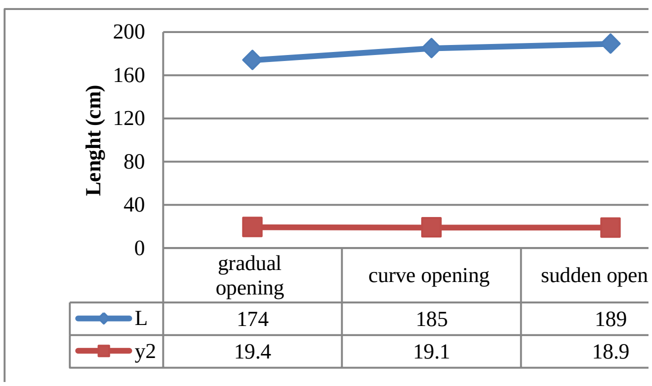


Figure 9. Graph of secondary jump length and depth at a flow rate of 41.4 l/s.

Table 3. Results of jump length in three-dimensional simulation.

N.	Opening shape	Flow rate	L/y1
1	Curved	27.8	26.80
2	Gradual	27.8	26.20
3	Sudden	27.8	24.60
4	Curved	41.4	37.80
5	Gradual	41.4	37.00
6	Sudden	41.4	34.80

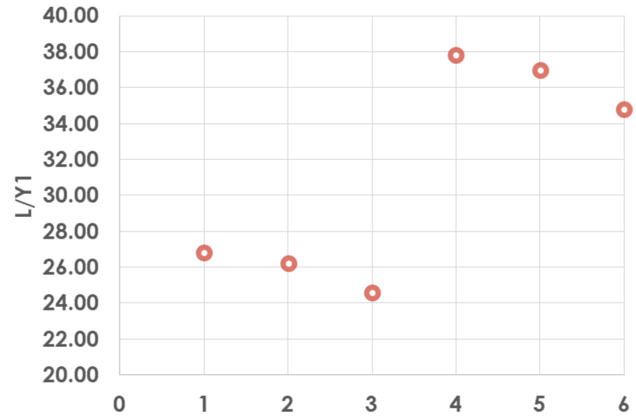


Figure 10. Survey diagram (L/Y1) in different models.

According to Table 3 and Figure 10, it can be well seen that at a flow rate of 27.8, the ratio of (L/y1) decreased from 26.8 to 24.6 by gradually and then abruptly changing from expansion. Also, in Flow rate 41.4, the ratio (L/y1) decreased from 37.8 to 34.8 with the gradual and then abrupt transformation of the expansion. The efficiency of classic stilling basins is 53.3%, and sudden stilling basins divergent is 70.7% according to Abdolmatin 2009 research. The results of mathematical modeling showed 71% efficiency for all three expansion forms. Depth after jumping in the stilling basins divergent modeled in Flow rate 27.8 and 41.4 l/s is 12 and 25% less than the classic basins, respectively. % Reduction of jump length compared to the classic mode in the parabolic, gradual, and sudden divergent stilling basins are 25.9%, 27.5%, and 31.8%, respectively. The L/Y1 dimensional length parameter was investigated for economic efficiency. It was found that sudden divergent stilling basin is more optimal due to shorter jump length.

Below, the FLOW-3D numerical model and the physical model are compared. The results are listed in Table 4. This comparison includes the average length and depth of the second jump in three discharges of 27.8, 36.2 and 41.4 l/s. The rest of the conditions are the same.

Table 4. Reviewing the results of the physical model and the FLOW-3D numerical model.

No	Test Number	Q(lit/s)	Test Type	RE	Opening Ratio	Y (CM)	FR1	Y2 (CM)	L (cm)
1	1	27.8	Experimental model	66609	0.5	3	4.27	14.9	121
2	2		Numerical Simulation	66609	0.5	3	4.27	15.1	123
3	3	36.2	Experimental model	90121	0.5	3	5.56	17.6	152
4	4		Numerical Simulation	90121	0.5	3	5.56	17.8	156
5	5	41.4	Experimental model	98154	0.5	3	6.36	19.1	169
6	6		Numerical Simulation	98154	0.5	3	6.36	19.4	174

According to Table 4, it can be seen that the simulation results with the FLOW-3D numerical model are close to the physical model. In the secondary depth, we see a difference of 1.36% and the jump 2.48% difference, which shows the mathematical model's high accuracy.

4.1 Neural network simulation results

Due to the high sensitivity of neural networks and fuzzy-neural system to the type of information used and the correlation of network inputs and subsequent outputs, apart from discussing the type of network and its use as a tool to generate artificial current, one must have a correct view of information, have access to and how to organize them for training and use of the network. Any kind of information with different time intervals does not necessarily lead us to the desired goal.

To calibrate the network, various excitation functions in the hidden layer and the output layer were used. Finally, by comparing the square of the squares, the errors in the test phase were determined. The linear axon function creates the lowest MSE in the network.

We used five different training algorithms in the hidden and output layers. By comparing the squares of the errors in the test phase, it was determined that the STEP algorithm creates the lowest amount of MSE in the network.

Seven different networks were created to calibrate the network and determine the optimal number of neurons in the hidden layer. Then the square of the errors was compared in the test phase. It was found that with five neurons in the hidden layer, the lowest amount of MSE is obtained. The calibration results are presented in Table 5. Table 6 summarizes the calibration results and the optimal neural network model of the research.

According to the problem and goals that we will have from the creation of the network, the type of statistics in terms of measurement accuracy and standard deviation values that can be discussed in the range can be discussed in issues related to hydraulics and hydrology. Overall, the accuracy of the test data will greatly affect the results. As mentioned, the incompleteness of a parameter or the out-of-bounds of one or more parameters is not effective or has little effect on the process of the artificial neural network, provided that the amount of data is multiplied. One of the properties of these networks is that with any data that the user gives them, provided that it is defined and specified, a result is obtained. The important point is that the occurrence of error or failure of out-of-frame results will cause it to be repeated many times in the process of the logical conclusion of the artificial neural network and will affect the reliability of the results. To create and build training patterns, the data arranged in the preprocessing stage were put together. Optimal artificial neural network means a network with high accuracy to produce outputs with high correlation data and squared squares with less error. In the present study, 70% of the laboratory data were used as training data for network training, 15% for testing and 15% for network validation. Input data matrix A matrix with dimensions of 158 * 4 includes information about the initial jump depth, flow rate, Froude number at the location of the initial jump depth and the ratio of the width of the primary section to the secondary section. Objective data matrix A 158*2 matrix contains information about the secondary jump depth and jump length.

Table 5. Calibration of the number of neurons in the network.

Number of Neuron	MSE Training	MSE Cross Validation	MSE test (10e-3)
1	0.132349423	0.181271728	93.11016788
2	0.05181275	0.072591356	8.469387315
3	0.019127996	0.03875391	9.01105544
4	0.056743949	0.082149969	11.63686939
5	0.055047935	0.078821795	6.508071849
6	0.051139935	0.081444374	33.71110049
7	0.0193921156	0.090301158	110.1410887

Table 6. Summary of neural network calibration results.

	MLP Model	
Hidden Layer No.	1	
Learning Algorithm	Hidden layer	Output Layer
	Step	Step
Transfer Function	LinearTanhAxon	LinearTanhAxon
Best PE in Hiden Layer	5	
Best Epoch	764	

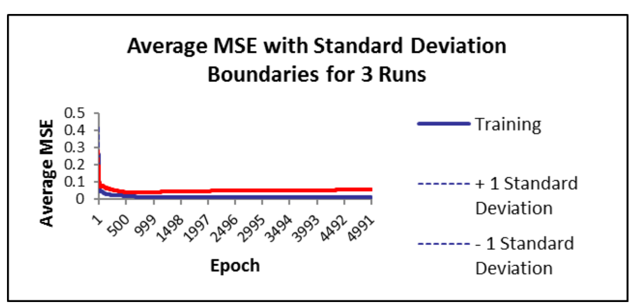


Figure 11. Graph of determining the number of repetitions of the appropriate training for the MLP model.

CONCLUSION

The optimized neural network model provides the least amount of output error. The above model can predict the jump length and secondary jump depth with MSE error of 7.0702e -7 and correlation coefficient of 0.093302. The values of the mentioned verification statistical indices show that the neural network can well estimate the selected output parameters (secondary length and depth of jump).

The results of the neural network are very close to the physical model. 3D simulation using Flow-3D software simulates a hydraulic jump with an average error of 2.41%.

The efficiency of divergent stilling basins was calculated to be 71%, which is higher than the classic stilling basins with an efficiency of 53.3%.

Depth after the jump in divergent stilling basins modeled at 27.8 and 41.4 l/s is 12 and 25% less than classic basins, respectively.

The% reduction in jump length compared to the classic mode in the divergent stilling basins parabolic is 25.9%, gradual 27.5% and sudden 31.8%.

Sudden divergent stilling basin has the best performance in terms of hydraulic flow parameters compared to other forms of expansion. The dimensionless scouring length parameter was investigated to evaluate economic optimality. It was found that sudden divergent stilling basin is more economically efficient due to shorter jump length.

If the effective input and output parameters are recorded during operation and the maintenance period, the neural network model can be prepared for the stilling basin. This model, with very low error, can help predict critical situations. This model can also be used with great precision in the design of similar basins. If the input data is planned correctly, the neural network model range can be increased.

3D numerical simulation using Flow-3D software with acceptable error for engineering design, all important output parameters in all control volume (speed changes in all three directions, the pressure at each point, the energy level at any point of control volume Etc.) shows. This is an important advantage of numerical simulation over the neural network model. Also, the cost of numerical simulation is much lower than physical modeling. Therefore, it is a more suitable option for designing numerical simulation engineering. Briefly for accuracy and application of numerical simulation by FLOW 3D software, high-efficiency stilling divergent basins, reduction of secondary depth and jump length in divergent stilling basins, longer jump reduction in sudden divergent stilling basins than gradual and curved ones and accuracy and application of neural network in divergent stilling basins calculations.

DECLARATIONS

Authors' contribution

All authors contributed equally to this work.

Conflict of interest

I hereby states that, there is no conflict of interest whatsoever with any third party.

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