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Determination of Optimal Dimensions of Concrete Gravity Dams Using LINGO11 Nonlinear Modeling

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ABSTRACT: Today, the economic considerations is an important factor in the selection of water resources projects, hence the attempt to minimize the economic costs of the project is essential. Typically, concrete gravity dams are more expensive than other dams, Thus by optimization of the dimensions of these types of dams, in order to reduce the volume of concrete, the project will be more economical. In this study, in order to optimize the dimensions of Koyna concrete gravity dam, the various forces that are applied on the dam were considered and were used from the nonlinear programming model in LINGO11 software. The results shown that, by optimization of the dimensions of Koyna dam, the volume of concrete used in dam construction is reduced and this project will be more economical. The Volume of concrete used in the construction of the Koyna dam in the existing state is equal to 3633m^3 and at optimal state is equal to 3551m^3. The result of optimization of dimensions of Koyna dam, shown that the volume of concrete was reduced about 82m^3. The results show that reduction of 2.26 percent of the volume of concrete used in dam construction, that is, economic in the high costs of dams' construction.

Keywords: Optimal Section, Concrete Gravity Dams, Nonlinear Programming, Koyna Dam

INTRODUCTION

Generally, dams are used as structures that store volume of water behind them. The dams are classified according to different criteria. One of the most important dams is gravity dams. In concrete gravity dams, the weight of dam is the main cause of the stability and balance against to driving forces such as earthquake force and uplift forces. Due to the high volume of concreting, with optimal design of dam dimensions, the volume of concrete and the cost of the project will be reduced (Varaei and Ahmadinadooshan, 2009). Due to the high costs of dam construction projects and considering that dams are large structures, with optimization of dimension and reduction in the volume of concrete used in the dam construction, the costs are reduced.

Some researches were conducted in the field of optimization of dimensions of concrete gravity dams and also using model LINGO, that some of will be introduced. In a study using LINGO model, Esmaeili and Baghalnejad (2007), obtained optimal utilization of reservoirs of the Khuzestan province dams. They concluded that the exploitation of reservoirs with extensive parameters needs to be optimized.

Riazi and Montazer (2008) in a study, evaluated use of surface and underground water resources of Qazvin irrigation network by using LINGO10 software. The results of their study shown that with changes in cropping patterns and the optimal combination of surface and underground water resources, with the optimal use of existing water resources, the amount of decrement in groundwater level in plain will be reduced, and the profits of the agricultural products will be saved. Varaei and Ahmadinadooshan (2009) in a study, optimized dimensions of concrete gravity dams using genetic algorithm and particle swarm algorithm and compared and evaluated the accuracy and speed of access to response. They concluded the large impact of amounts of particle swarm algorithm in convergence.

Shahbazi and Sadeghian (2009) in their research with development of a linear programming (LP) model, optimized the reservoir of Karun 4 hydroelectric Dam, with the goal of maximizing primary and secondary energy, according to the annual costs in order to generate more profits. The results of this model shown that, this model in terms of the quality had the results of better than other models. So that other models required to more computational effort and longer period of time.

Karimipashaki and Saremi (2010) in a study investigated optimal management for use of irrigation networks using LINGO10 linear programming model. The objective function of the problem is consists of determination of the volume of water that is delivered from the reservoir to the downstream networks, So that to supply of the needs, the total of project costs must be least. They developed their model using LINGO10 programming model and presented the patterns for optimal utilization, particularly in water scarcity situations and drought.

Kazemi et al. (2013) in a study obtained optimal dimensions of concrete gravity dams using GAMS optimization model and MP5 simulation model. They optimized dimensions of dam by developing optimization model in GAMS software and connected this optimization model to M5 model. The results of the application of the methodology proposed by Kazemi et al in Pine Flat dam in Colorado of United states shown that decrease in volume of concrete used in the optimal design towards actual design of the dam. Cao et al. (2007) in a study optimized water networks using genetic model

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development and they used LINGO linear programming model (LP) for Systems of contaminant of water resources, and they compared these results with results of Genetic Algorithm.

Ghassan et al. (2013) in their research, investigated optimization and simulation of models that has been used for the analysis of water resource systems. They used the linear programming models (LP) to estimate the maximum of allowed utilization in the single-reservoir system. One of the models was optimized completely and other models were optimized simply, Ghassan et al. (2013) concluded from this model in solving the problem.

The aim of this study is optimization of dimensions of Koyna concrete gravity dam, by considering the various forces that are applied on the dam by using LINGO11 nonlinear programming model. In this study, the aim is to ensure the stability of the dam against sliding and overturning and tension control in the dam structure and by considering the forces that are applied on the dam, that are consist of earthquake force on the dam structure and in the dam reservoir, uplift force, Forces that are caused by water pressure, dam's weight and sediments pressure, dimensions of Koyna concrete gravity dam were optimized. For this purpose, generally, total section of this concrete gravity dam is designed, that is consist of eight design variables. Volume of concrete that has been used in the dam construction was introduced as the objective function and the optimization problem was simulated and designed under tension constraints, sliding and overturning.

MATERIALS AND METHODS

Concrete gravity dams

Concrete Gravity dams are dams that dam equilibrium is established with the help of external forces such as water pressure forces, wave pressure, sediment pressure, uplift pressure and dam weight. In gravity dams, dam weight is important factor in stability of structure and to balance of driving forces such as earthquake and uplift forces. Secondly, in the dam construction is used from the concrete. The cost of gravity dam's construction is more than other dams. So if we optimize the dimensions of concrete gravity dams by considering all the conditions of stability and appropriate safety factors and standard, the volume of concrete used in the dam construction are reduced, thus the project will be economic (Arzide, 1983).

Introducing the Study of Dam (Koyna Dam)

The height of Koyna concrete gravity dam is 103.2m and has been located on the Koyna river in India country. This dam is in the range of 73 degrees and 45 minutes east longitude and 17 degrees and 24 minutes northern latitude. The purpose of the construction of this dam is flood control. The height of flood elevation and normal elevation in upstream are 103 and 91.75 meters respectively. Koyna concrete gravity dam was designed in 1956 and exploitation of this dam was begun in 1964. Table 1 presents the general characteristics of Koyna concrete gravity dam. Also in Table 2, has been presented Geometry and hydraulic dimensions of Koyna dam (Calayir and Karaton, 2005).

Table 1. General characteristics of Koyna concrete gravity dam

Dam name	Crest length (m)	Reservoir volume (MCM)	crest elevation (m)	free board(m)	Type of Spillway	Year of exploitation
Koyna	14.8	2797.4	1113	11.45	Ogee	1964

Table 2. Geometry and hydraulic dimension of Koyna dam

Variable	**Volume**	**b_1**	**b_2**	**b_3**	**b_4**	**b_5**	**b_6**	**b_7**	**b_8**	**H_u**	**H_d**	**H_{uf}**	**H_{df}**
Amount	3633	1.63	14.8	5.6	48.19	39	36.5	66.5	64	91.75	0	103	0

That H_u= water elevation in the normal state in upstream of dam, H_d= water elevation in the normal state in downstream of dam, H_{uf} = water elevation in the flood state in upstream of dam and H_{df}= water elevation in the flood state in downstream of dam. Also b_1 to b_8 variables are dimensions of Koyna concrete gravity dam that has been presented in Figure 1.

Introducing the LINGO11 Model

LINGO software has been designed in 1994 to solve optimization problems by Professor Wayne L. Winston. This software has a high potential to solving variety of programming problems. This model has ability of solving to the methods of linear, nonlinear, integer and the combination, considering the objective function and constraints. To optimize by LINGO model, initially we introduce objective function and decision variables in optimization problem. Then we define the constraints of problem. By considering these constraints, after that we run the program and then are obtained decision variables and the objective function (Manual LINGO software, 1998).

Problem modelling

In order to optimize the dimensions of concrete gravity dam, the sections are considered that are consist of geometric parameters (b_1 to b_8) as design variables, and other geometric parameters depend on these variables that has been defined. Sectional area of the dam is the objective function that the aim is to reduce this area. Then all the forces and torques were obtained by the design variables. And normal tension values were calculated at the upstream of dam and also were calculated the safety factor against overturning and sliding. Then the relevant equations were developed in LINGO11 software. To investigation of this forces that are applied on the gravity dam, the dam is considered by unit length. In Figure 1 has been presented two-dimensional view of the geometry dimensions of the Koyna concrete gravity dam.

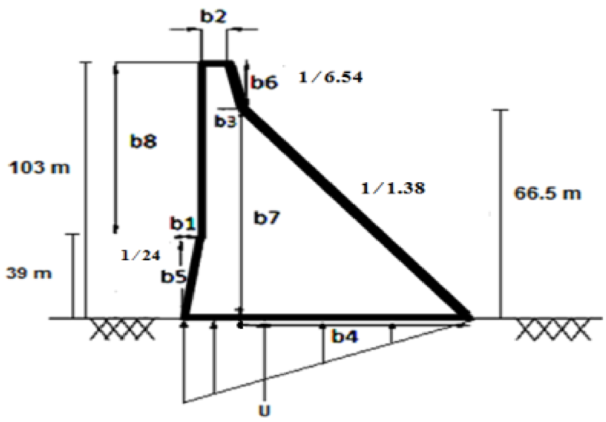


Figure 1. Two-dimensional view of the Koyna concrete gravity dam

Generally, the forces that are applied on concrete gravity dams are consist of the force of dam weight, hydrostatic force of water pressure, the force of water weight in upstream of dam (on the inclined side of upstream), uplift force, the force of sediments pressure in dam reservoir, ice force, wave force, wind force and earthquake force (on the dam structure and the dam reservoir). Because of these forces are not applied on the dam structure simultaneously, therefore, in this study the critical composition is considered to analyze the stability of the dam. that are consist of force of dam weight, hydrostatic force of water pressure, the force of sediments pressure, uplift force and the earthquake force (on the dam structure and the dam reservoir) (Goldberg, 1989).

USBR proposed loading combinations

1- Typical loading: In this state are considered effect of dead load, hydrostatic force of water at normal and design elevation (spillway crest) and sediments forces (If there are), uplift force and ice force, simultaneously. If there are loads of caused by temperature changes, are applied minimum temperatures.

2-Abnormal loading: In this type of composition are considered effect of dead load, the maximum of hydrostatic force of water in upstream elevation and the sediments forces (If there are), uplift force and minimum of temperatures, Simultaneously.

3-Extra loading: In Extra loading, all loads of typical loading affect, in addition to a reasonable maximum of the earthquake force (Design of small Dams, 1987; Gravity Dam Design, 1995).

In this study, loading combinations are considered in Extra loading state. The forces that are applied on the dam are divided to two groups: Forces in the direction of stability and instability. The forces that act on the stability of the dam are consist of the force of dam weight, the vertical component of hydrostatic force of water pressure, the force caused by sediments pressure that are applied on the dam vertically. Also the forces that act against the stability of the dam that are consist of the horizontal component of hydrostatic force of water pressure, Uplift force, Earthquake force (on the dam structure and the dam reservoir) and sediments pressure that are applied on the dam horizontally.

The force of dam weight is obtained according to the weight of materials used in dam. The concrete specific weight (γ_c) is considered 22.5 to 23.5 KN/m^3 roughly. To calculate this force, according to the upstream and downstream slopes and the central portion of the dam, the dam longitudinal sections are divided to a series of triangles and rectangles, and then the area of the dam is calculated. Calculations are done for one meter wide. Therefore, it is necessary to be multiplied area in concrete specific weight (γ_c) (Abrishami, 2001).

$$w = \left[\left(\frac{1}{2}\times b_1\times b_5\right)+\left(b_2\times(b_8+b_5)\right)+\left(\frac{1}{2}\times b_3\times b_6\right)+\left(b_3\times b_7\right)+\left(\frac{1}{2}\times b_4\times b_7\right)\right]\times\gamma_c \qquad (1)$$

That w=dam weight, γ_c=the concrete specific weight and b_1 to b_8 are the sizes that has been presented in Figure 1.

Water pressures that are applied on the dams are calculated from the statute of hydrostatic pressure (Charter of pressure). Water specific weight is considered 9.81 KN/m^3. According to non-vertical slope in upstream, hydrostatic pressure in upstream are consist of horizontal and vertical components. Horizontal component acts on the opposite of direction of sustainability and vertical component acts on the direction of sustainability. Vertical component of this pressure is water weight on the slope. To apply the pressure in the downstream, it is necessary to be considered the minimum of possible elevation. For this reason, this force is caused the balance in dam. The height of water is variable in upstream, usually water elevation in the state of overflow or exploitation and the flood elevation is more important and is applied in the calculations. The point of impact of this force is considered in two-thirds of the height from the water surface. Equations 2 and 3 calculate horizontal hydrostatic pressure and water weight, respectively.

$$Fh_h = \frac{1}{2}\times\gamma_w\times h^2 \qquad (2)$$

$$Fh_V = \gamma_w\times V_W \qquad (3)$$

That h= Normal elevation of water in dam reservoir, γ_w=water specific weight and v_w= the volume of water on the upstream slope.

Because of alluvial rivers, sediments are entered in the reservoirs of dams as bed and suspended load. Sediments with reducing in their speed, collapse toward dams. In the calculation of this force is assumed that Sediments are saturated and non-adherent completely. Uplift pressure is completely and there is its internal friction. Sediments pressure is applied on the dam structure horizontally and vertically (If the dam structure is inclined), this pressure is calculated and effect point of the horizontal pressure is in the one-third of accumulated sediments height from the floor. The effect point of the vertical pressure is in the gravity center of accumulated sediments on the slope. This force is unstable factor in the calculations. Equations 4 and 5 calculate the sediments forces horizontally and vertically.

$$P_{SH} = \frac{1}{2}\times\gamma_s\times h_s^2\times\frac{1-\sin\varphi}{1+\sin\varphi} \qquad (4)$$

$$P_{SV} = \frac{1}{2}\times\gamma_s\times h_s^2\times\tan\theta \qquad (5)$$

That h_s= sediments height, φ=Angle of internal friction of sediments, γ_s=volumetric weight of sediments and Θ= Angle of inclination in the upstream of dam.

According to the permeability of the dam foundation and seepage and the pressure difference between the upstream and downstream of dam, the upward force is applied on the bottom part of the dam that is called uplift force. To calculate this force is used from the linear distribution of pressure in this part. Equation 6 calculates uplift force.

$$F_U = \frac{1}{2} \times \gamma_w \times (h + h') \qquad (6)$$

That h and h′ are uplift elevation in the tiller and heel of the dam. Usually are considered equal to half of height of water (Beirami, 1997).

The dams must be designed so that resist against possible Earthquake in their useful lifetime. Earthquake acceleration is determined as a multiple of the acceleration of gravity (g). Then it be considered earthquake acceleration equal to αg, α is the coefficient of earthquake, that this coefficient is considered the minimum 0.02 to maximum 0.3 and usually is considered between 0.1 to 0.15 for large dams in earthquake-prone areas. The earthquake force is function of factors that are consisting of the magnitude of earthquake, dam weight and the amount of elasticity of the dam constituent materials. As regards the waves caused by earthquake move in any direction, for simplicity of analysis, is considered two horizontal and vertical components for earthquake acceleration. That usually vertical acceleration is Two-thirds of the horizontal acceleration, also be reduced by half.

The earthquake force is divided to two groups that are earthquake force caused by dam weight and earth-quake force in the dam reservoir, that are calculated in horizontal and vertical states. In this study, these forces were considered for tension and stability calculations.

Optimization using nonlinear programming

Optimization problems have three main characteris-tics that are consist of objective function, the constraints and decision variables. In this study, in the optimization problem of dimensions of concrete gravity dam, and after surveys conducted, these functions have been defined as follows:

The objective function of problem is to minimize the dam area or volume of concrete used in the dam construction. That has been presented in the following relations.

$$C = \text{Min } V(x) \qquad (7)$$

$$C = \text{Minimize}\left[\left(\frac{1}{2} \times b_1 \times b_5\right) + (b_2 \times (b_8 + b_5)) + \left(\frac{1}{2} \times b_3 \times b_6\right) + (b_3 \times b_7) + \left(\frac{1}{2} \times b_4 \times b_7\right)\right] \qquad (8)$$

That V(x) = volume of concrete, b_1 to b_8 are dimensions of Koyna concrete gravity dam. Also in this study, decision variables are dimensions of Koyna concrete gravity dam that are consist of b_1 to b_8 variables.

To evaluate the stability of dams, three types of stability must be controlled that are consisting of Stability against sliding, tension or vertical fatigue on the dam structure and also stability against overturning. These cases, in this study are the constraints. In the following, they have been introduced.

Safety factor against sliding

This coefficient is defined in equation 9, that b= the Length of base in the study level, σ= allowable shear tension in the cutting surface. Allowable shear tension for concrete is about one-fourth of shear strength or one-twentieth of its compressive strength.

Allowable shear tension for concrete is considered to be between 7 to 14 kg/cm^2. The coefficient of static friction f' is changed between 0.65 to 0.75, that is considered to be equal to 0.7. According to the USBR standard, for small dams is considered SFF≥4, if the dam breach create human and financial Damages (Design of small Dams, 1987). The tensions that are created in the dam structure must be in a certain range and the tension should not be exceeded from allowable tension at any point of the dam (Gravity Dam Design, 1995).

$$SFF = \frac{f' \sum F_V + b\sigma}{\sum F_H} \qquad (9)$$

The tension or vertical fatigue on the dam structure

To study of vertical fatigue on the dam structure in upstream and downstream, these relations have been presented.

$$\sigma_U = \frac{\sum F_V}{b} - \frac{6 \sum M_o}{b^2} \qquad (10)$$

$$\sigma_d = \frac{\sum F_V}{b} + \frac{6 \sum M_o}{b^2} \qquad (11)$$

In these relations σ_u and σ_d are vertical fatigue on the dam structure in upstream and downstream, respectively, $\sum M_o$= Total of torques of forces that are applied on the dam structure toward the center of surface, and $\sum F_v$= Total of the vertical forces. It should be mentioned that in this study, the forces in the direction of gravity force is considered positive and the forces in contrast to gravity force is considered negative. Also the torques in clockwise are positive and in contrast are considered negative. For stability of the dam against the vertical fatigue, σ_d and σ_u should be positive in the states of full or empty of dam reservoir.

This amount should not be exceeded from the allowable compressive strength. Usually the compressive strength of concrete for dams is considered in the range of 140 to 350 kg/cm^2 (Design of Small Dams, 1987).

The Stability against overturning

If the resistant torques in the toe of the dam be about 1.5 to 1.7 times more than total overturning torques towards the same point, the dam will remain stable against overturning. Its relation has been presented in the following (Design of small Dams, 1987).

$$SF_O = \frac{\sum M_R}{\sum M_o} = 1.5 - 1.7 \qquad (12)$$

That M_R= Torque of resisting forces, M_O= Torque of driving forces on the dam.

After introduction of the optimization parameters that are consist of objective function, decision variables and constraints, with preparation of model in LINGO software, the model developed and simulated. and conclusions are consist of objective function and decision variables.

CONCLUSION AND DISCUSSION

After optimization of dimensions of Koyna concrete gravity dam using nonlinear programming in LINGO11 software, the optimal dimensions of the dam that are decision variables in optimization problem, were calculated. The results have been presented in Table 3.

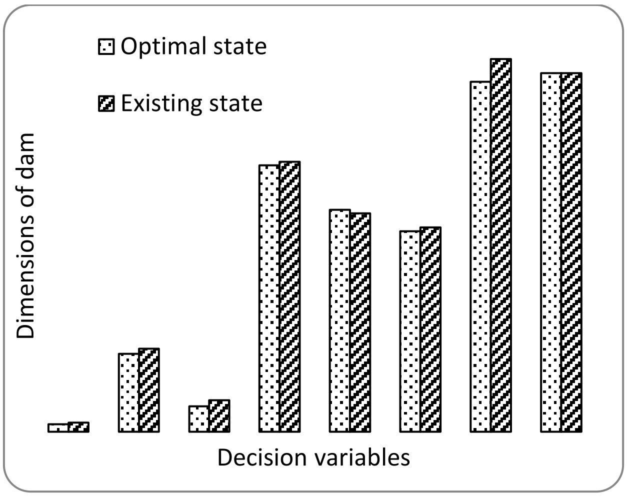
Table 3. Optimal dimensions of Koyna concrete gravity dam

Parameter	$V(m^3)$	$b_1(m)$	$b_2(m)$	$b_3(m)$	$b_4(m)$	$b_5(m)$	$b_6(m)$	$b_7(m)$	$b_8(m)$
Existing dimensions	3633	1.63	14.8	5.6	48.19	39	36.5	66.5	64
Optimal dimensions	3551	1.62	14.8	5.59	48.17	36.32	26.62	64.67	66.98

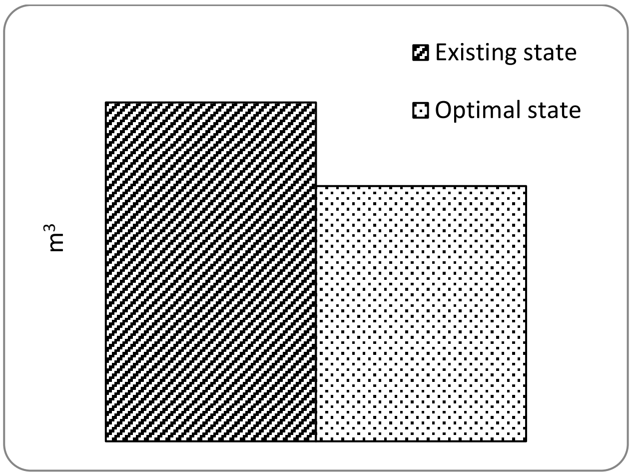
The volume of concrete used in the dam construction is equal to 3633m^3 in state of existing dimensions and was equal to 3551m^3 in state of optimal dimensions. After optimization of dimensions of Koyna concrete gravity dam, the volume of concrete used in the dam construction has been reduced about 82 m^3. Also the costs are decreased. According to the results of table 3, the maximum of reduction is related to b_6 value and minimum of reduction is related to b_1 value. b_2 value has been fixed. It should be mentioned that the amount of b_8 value has been increased by optimization of dimensions of Koyna concrete gravity dam. In graphs 1 and 2, has been compared the dimensions of Koyna concrete gravity dam and the volume of concrete used in the dam construction, before and after optimization.

Also in graph 1 has been compared dimensions of Koyna concrete gravity dam, before and after optimization. The results show that some dimensions have been reduced and also the volume of concrete used in the dam construction. So the costs of project are reduced. The results show that the volume of concrete used in the construction of Koyna concrete gravity dam has been reduced 2.26 percent, after optimization.

For the proof of model performance, the results of this study were compared with the results of other studies that had been optimized by LINGO model. The results of this study that show reduction in the volume of concrete gravity dam, corresponded to the results of study by Esmaeili and Baghalnejad (2007), they have determined optimal utilization of reservoirs of the Khuzestan province dams by using LINGO model. The optimization of the volume of concrete used in construction of concrete gravity dam and reduction in the costs of projects in this study, corresponded to the results of study by Kao et al. they optimized water networks by using genetic model development. They had used LINGO linear programming model (LP) for systems contaminant of water resources and compared these results with results of Genetic Algorithm. Also, the results of this study were compared with the results of study by Varaei and Ahmadinadooshan (2009). That they had optimized the dimensions of concrete gravity dams by using other optimization methods. The results show that in optimization of dimensions of dam by other methods, some of dimensions values of dam have been fixed.



Graph 1. The dimensions of Koyna concrete gravity dam in the state of existing and optimal dimensions (m)



Graph 2. Comparison of the volume of concrete used in the dam construction in existing and optimal state

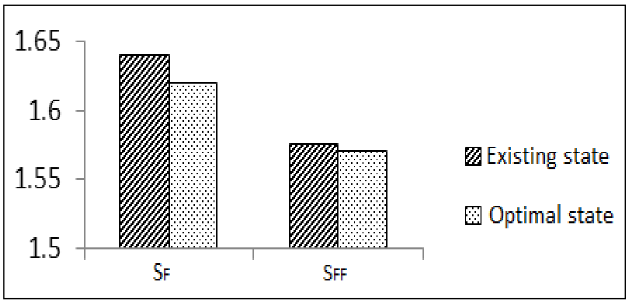
After determination of optimal dimensions of dam, for the proof of model Performance, was calculated the stability against overturning, stability against sliding and the tension on the dam by using the values of optimal dimensions. The results of calculations have been presented in Table 4.

Table 4. Stability against overturning, sliding and the tension on the dam structure (in existing and optimal state)

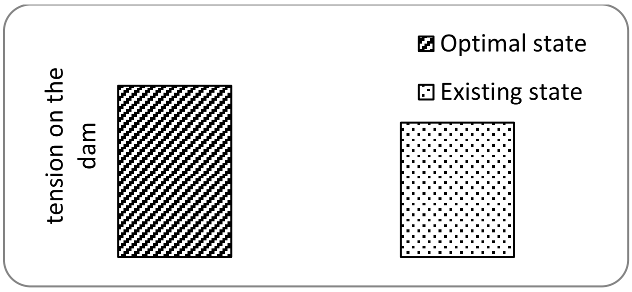
Extra Loading	S_F	S_{FF}	σ_U
Existing state	1.64	1.58	90.39 (ton /m^2)
Optimal state	1.62	1.57	115.18(ton/m^2)

That S_F=Safety factor against overturning, S_{FF}= Safety factor against sliding and σ_U= tension on the dam.

The results show that the constraints values have appropriate and dependable values. According to standards that mentioned, Koyna concrete gravity dam is stable, with optimal dimensions. In graphs 3 and 4 has been presented Safety factor against overturning, Safety factor against sliding and the values of the tension on the dam in existing and optimal states.



Graph 3. Safety factor against overturning and sliding in Existing and Optimal states



Graph 4. The tension on the dam in Existing and Optimal states

CONCLUSION

In this study, Koyna concrete gravity dam was optimized by using LINGO11 nonlinear modeling. The results show reduction in the volume of dam and reduction in the volume of concrete used in dam construction, that is leads to reduction in the costs of performance.

After determination of decision variables in the optimization problem, that are dimensions of concrete gravity dam, for the proof of model performance, safety factor against overturning, safety factor against sliding was calculated. Also, the values of tension on the dam were calculated. The results show that Koyna concrete gravity dam was stable in optimal state.

Suggestions

1. With data collection, this model is implemented for the other dams and its result be compared with actual values.

2. It is recommended that be used from meta-heuristic algorithms for optimization of dimensions of this dam and its result is compared with this study.

3. The results of this study are compared with the results of laboratory models.

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