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Effective Moment of Inertia of Single Spanned Reinforced Concrete Beams with Fixed Beam-Column Joints

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ABSTRACT

Of prime importance in the determination of the deflection of beams is the calculation of the moment of inertia (I) of the beam, the value of which changes along the span length from "I_g" for uncracked sections to "I_{cr}" for cracked sections. From literature, many experimental works have been carried out on simply supported beams with varying concrete characteristic strengths and percentages of reinforcement. However, none was on beam with fully or partially restrained ends. Hence the focus of this research work is to determine the effective moment of inertia (I_e) of a cracked L-section of reinforced concrete beam with full end restrained. Three existing models for determining "I_e" were used in the estimation of the deflection of the beam, and these existing models were modified in order to get a proposed model that gives a more accurate prediction of the deflection. At service load of 9.81 kN/m, the estimated deflections using the values of I_e from the existing three models and the proposed model were 2.01 mm, 9.81 mm, 2.68 mm and 8.37 mm respectively, while the actual deflection was 8.14 mm. From these results, the proposed model predicts more accurately the deflection of the L-beam than the three existing models, however, it is recommended that further research should still be carried out on reinforced concrete beams with fixed beam-column joints, in order to get a model that can predict more accurately, the effective moment of inertia "I_e" for other types of beams such as rectangular and T-beams.

Keywords: Deflection, Uncracked Moment of Inertia; Cracked Moment of Inertia; Effective Moment of Inertia; Cracking Moment; Elastic Modulus of Concrete.

INTRODUCTION

Reinforced concrete structures must satisfy both the ultimate and serviceability limit states. The serviceability limit states of crack widths, deflections and excessive vibrations are of prime importance. Historically, deflections and crack widths have not been a problem for reinforced concrete building structures (Wight and MacGregor, 2009). The introduction of high-strength concrete, high-strength reinforcing bars, coupled with more precise computer-aided design softwares, the limit-state serviceability design, has resulted in lighter and more material-efficient structural elements and systems. This in turn has necessitated better control of short-term and long-term behavior of concrete structures at service loads (ACI 435R, 2000).

In practice, deflection control is based on the deemed – to – fit provisions of the codes (Nkuma, 2013). However, damages such as cracks have been noted on partition walls of buildings resulting from excessive deflection of slabs and beams even when the serviceability requirements for deflection based on these deemed-to-fit provisions of the code were satisfied (Nkuma, 2013). Hence there is the need to estimate the expected deflection at service loads which will be compared with the permissible deflection from the codes. One of the major factors that affects the deflection of flexural members is the effective moment of inertia.

The deflection of a flexural member is a function of the support conditions, applied load and span, and the flexural rigidity of the member. The majority of the building codes do not concern themselves with computations of deflections but rather with attempting to provide minimum values of flexural rigidity. Deflection of reinforced concrete flexural members is controlled by reinforcement ratio limitations, minimum thickness requirements, and span/deflection ratio limitations.

The minimum thickness provisions of American Concrete Institute (ACI 435R, 2000) for deflection control are contained in Table 2.4 of ACI 435R (2000), while the basic span-effective depth ratios provisions of the BS 8110 are contained in Table 3.9 of BS 8110-1 (1997). The allowable computed deflections specified in ACI 318 (2005) for one-way systems are reproduced in Table 2.5 of ACI 435R (2000), where the span-deflection ratios are provided for a simple set of allowable deflections.

Before cracking, the entire cross section is stressed by load. The moment of initial of this section is called the un-cracked moment of inertia, which is usually, the gross un-cracked transformed moment of initial for the concrete section(I_g). As the load is increased, flexural cracking occurs when the moment exceeds the cracking moment. The corresponding moment of inertial for this cracked section is referred to as the cracked moment of inertial(I_{cr}). The deflection of a beam is calculated by integrating the curvatures along the length of the beam (Wight and MacGregor, 2009). For an elastic beam, the curvature, $\frac{1}{r} = \emptyset$, is calculated as $\emptyset = \frac{M}{EI}$, where EI is the flexural stiffness of the cross section. When the integration is completed it can be seen that the deflection of a member is a function of the span length, support or end conditions, the type of loading and the flexural stiffness, EI. In general the elastic deflection for non-cracked members can be expressed as Eq. (1):

$$\delta = k\frac{Ml^2}{E_c I_g} \qquad Eq.(1)$$

Where, k is a factor depending on the degree of fixity of the support, M is maximum moment, l is clear span length, E_c is elastic modulus of concrete and I_g gross moment of inertia of the section (ACI 435R, 2000). The elastic modulus of concrete E_c can be estimated using equation (2) below (Wight and MacGregor, 2009):

$$E_c = 57{,}000\sqrt{f_c^1} \quad (psi) \qquad Eq.(2a)$$

$$E_c = 4{,}725.64\sqrt{f_c^1} \quad (N/mm^2) \qquad Eq.(2b)$$

For the reinforced concrete beam, however, three different values of I must be considered depending on section condition. When the section is un-cracked, the value of I is equal to I_g. The value of $I = I_{cr}$ is used when the beam section is fully cracked. For the beam with partially cracked section the value of Imust be taken as I_e (Akmaluddin and Thomas, 2006).

Branson (1965) used Eq. (3) to express the transition from I_g to I_{cr} that is observed in experimental data:

$$I_e = \left(\frac{M_{cr}}{M_a}\right)^m I_g + \left[1 - \left(\frac{M_{cr}}{M_a}\right)^m\right] I_{cr} \le I_g \qquad Eq.\ (3)$$

Branson's effective moment of inertia expression (Equation 3), which averages the moments of inertia of the un-cracked and fully-cracked portions of a concrete beam, is adopted by ACI 318 (2005), which set the value of m to 3 to obtain an average moment of inertia for the entire span of a beam and this is expressed as Eq. (4).

$$I_e = \left(\frac{M_{cr}}{M_a}\right)^3 I_g + \left[1 - \left(\frac{M_{cr}}{M_a}\right)^3\right] I_{cr} \le I_g \qquad Eq.\ (4)$$

Where:

M_{cr} = Cracking moment; M_a = Maximum service load moment (un-factored) at the stage for which deflections are being considered; I_g = Gross moment of inertia of section; I_{cr} = Moment of inertia of cracked transformed section

And

$$M_{cr} = \frac{f_r I_g}{y_t} \qquad Eq.\ (5)$$

$$f_r = 0.623\,\gamma_c\sqrt{f_c^1}\ MPa \qquad Eq.\ (6)$$

Where:

f_r = modulus of rupture; f_c^1 = concrete grade; $\gamma_c = 1.0$ for normal density concrete (2325 to 2400 kg/m^3), 0.85 for semi low-density (1765 to 2325 kg/m^3) and 0.75 for low-density concrete (1445 to 1765 kg/m^3).

For continuous members, ACI 318 (2005) stipulates that I_e, may be taken as the average values obtained from Eq. (7)) for the critical positive and negative moment sections. For prismatic members, I_e, may be taken as the value obtained at mid-span for continuous spans (ACI 435R, 2000). If the average effective moment of inertia I_e is to be used, then according to ACI 318 (2005), the following expression should be used:

$$I_e = 0.5 I_{e(m)} + 0.25(I_{e(1)} + I_{e(2)}) \qquad Eq.\ (7)$$

Where the subscripts *m*, *1*, and *2* refer to mid-span, and the two beam ends, respectively.

The value of I_e, can also be affected by the type of loading on the member (Al-Zaid, 1991), i.e. whether the load is concentrated or distributed. Furthermore, Al-Zaid *et al* (1991) experimentally showed that the power m in the effective moment of inertia expression is affected by the loading conditions of a beam and the load level (M_a/M_{cr}).

In their study, Al-Shaikh and Al-Zaid (1993) revealed that Branson's model underestimated the effective moment of inertia of all test specimens. The underestimation of I_e was approximately 30% in the case of a heavily reinforced member and 12 % for a lightly reinforced specimen. Beyond the previously observed behaviour of a reinforced concrete member subjected to a mid-span concentrated load (Al-Shaikh and Al-Zaid 1993), it is obvious that reinforcement ratio affects the accuracy of Branson's model especially when the member is heavily reinforced and that the value of m decreases as the reinforcement ratio (ρ) of a concrete beam increases. Accordingly, they proposed the following equation for m:

$$m = 3 - 0.8\rho \qquad Eq.(8)$$

Nonetheless, different studies (Scanlon et al., 2001; Gilbert, 1999 and Gilbert, 2006) indicated that Branson's model constantly overestimates the moments of inertia of reinforced concrete beams with low reinforcement ratios ($\rho < 1\%$), which causes underestimation of the deflections. Bischoff (2005) found out that the underestimation of the moments of inertia and deflections of lightly-reinforced concrete beams by the Branson's approach is caused by the overestimation of the tension stiffening of concrete. According to the analytical study carried out by Bischoff (2005), the tension-stiffening component in Branson's method depends on the applied load level (M_a/M_{cr}) and on the ratio of the gross moment of inertia to the cracked moment of inertia (I_g/I_{cr}) of the beam, which varies inversely with the reinforcement ratio (ρ). Branson's expression provides accurate estimates for reinforced concrete beams with reinforcement ratios greater than 1%, which corresponds to an I_g/I_{cr} ratio of 3. For lower reinforcement ratios (I_g/I_{cr} > 3), the member response estimated by Branson's approach is stiffer than the actual response, resulting in the under prediction of the deflections (Kalkan, 2013).

Bischoff (2005) presented the application of the method to the in-plane bending behavior of reinforced concrete beams and developed the following effective moment of inertia expression, which is a weighted average of the flexibilities of the un-cracked and cracked portions of a reinforced concrete beam:

$$\frac{1}{I_e} = \left(\frac{M_{cr}}{M_a}\right)^m \frac{1}{I_g} + \left[1 - \left(\frac{M_{cr}}{M_a}\right)^m\right] \frac{1}{I_{cr}} \qquad Eq.\ (9)$$

A value of *2* was proposed for the power m in Equation (9), based on the deflection equation given in Eurocode 2 (CEN, 2002). The use of $m = 2$ assures that the tension-stiffening contribution in the model is only dependent on the applied load level (M_a/M_{cr}), as explained by Bischoff (2005) and Bischoff (2007), in detail. Consequently, the tension-stiffening model becomes independent from the gross-to-uncracked moment of inertia ratio (I_g/I_{cr}) and the reinforcement ratio (ρ) of the beam.

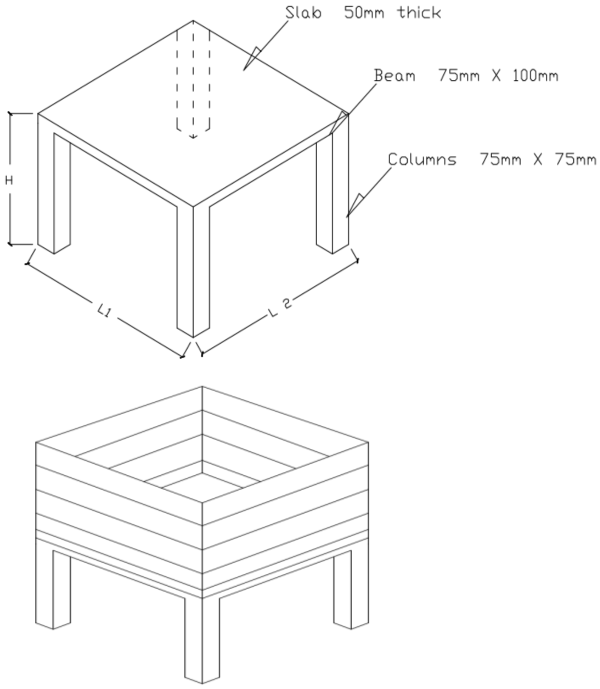
In their work, Ammash and Muhaisin (2009), presented a new form of the effective moment of inertia model by enhancement of Branson's model, taking into account the effect of several factors such as type of loading, shear deformations, reinforcement ratio. The models resulted from their studies were compared with (experimental results, Branson's model results, and results of other models). The results of the model give best agreement with experimental results than Branson's and the other models. The results showed that the effective moment of inertia reduced by about 27% for span to depth ratio of (20 to 5) due to shear deformation effects and gives good agreement with the experimental results for all types of cross section.

Also, Ammash et al. (2018), proposed a model for estimating deflection. This model takes into consideration parameters such as grade of concrete, loading conditions and type of reinforcement. The results of the proposed model showed a better agreement with the experimental studies when compared to ACI equation and other models from literature. The maximum difference in deflection results from the proposed model and actual deflection from experimental work was between 1% and 10%.

From literature, extensive research work have been carried out on the determination of effective moment of inertia for simply supported beams, however none was on beam with fully or partially restrained ends. Hence the aim of this research is to determine the effective moment of inertia (I_e) of a cracked L-section of reinforced concrete beam with beam-column joint fixed.

2.0 MATERIAL AND METHODS

2.1 Materials. The materials used for this work is a square, single panel, reinforced concrete space framed model with beam-column joint fixed, constructed from micro-concrete (using sand from borrowed pit), loading box, laterite as the loading material, and dial gauges. Figure 1 shows a typical model. The loading box which measured 1m x 1m x 3.0 m was placed on top of the model, and three dial gauges were placed at the centre of the slab and centres of two adjacent beams. Manually, known weight of laterite using head-pan was poured into the loading box and readings of the dial gauges for deflections of the beams and slab were taking at every 1.84 kN load of laterite. The process continued until collapse occurred.



b)

a)

Figure 1: Square space framed model. (a): Schematic figure of the Square RC (Space Framed Model); (b): Schematic figure of the Square RC Space Framed **(**Model with the Loading Box)

2.1.1 Cement. Ordinary Portland Cement (OPC) obtained from Larfarge Cement, Ewekoro, Abeokuta, Ogun State of Nigeria, was used in this study. The OPC used complies with Type I Portland cement as in ASTM C150-02a (2002).

2.1.2 Soil. The fine aggregate was collected from a borrowed pit from Akure metropolis, while the coarse aggregate was purchased from JCC Quarry, along Akure-Owo road. From Figure 2, the coefficient of curvature Cc for sand is 2.90 and the coefficient of uniformity C_u for sand is 1.06. These values indicate that the sand is well graded, since it is within the satisfactory range of 2 and 3 for coefficient of curvature, as specified by the British Standard Institution (BS 812-103.1, 1985).

2.1.3 Water. For this experimental study, tap water was used to produce the space framed structures. The water/cement (W/C) ratio used for the research work was 0.55.

2.1.4 Reinforcement. Mild steel reinforcement with characteristic strength of 250 N/mm^2, was used for this experimental work.

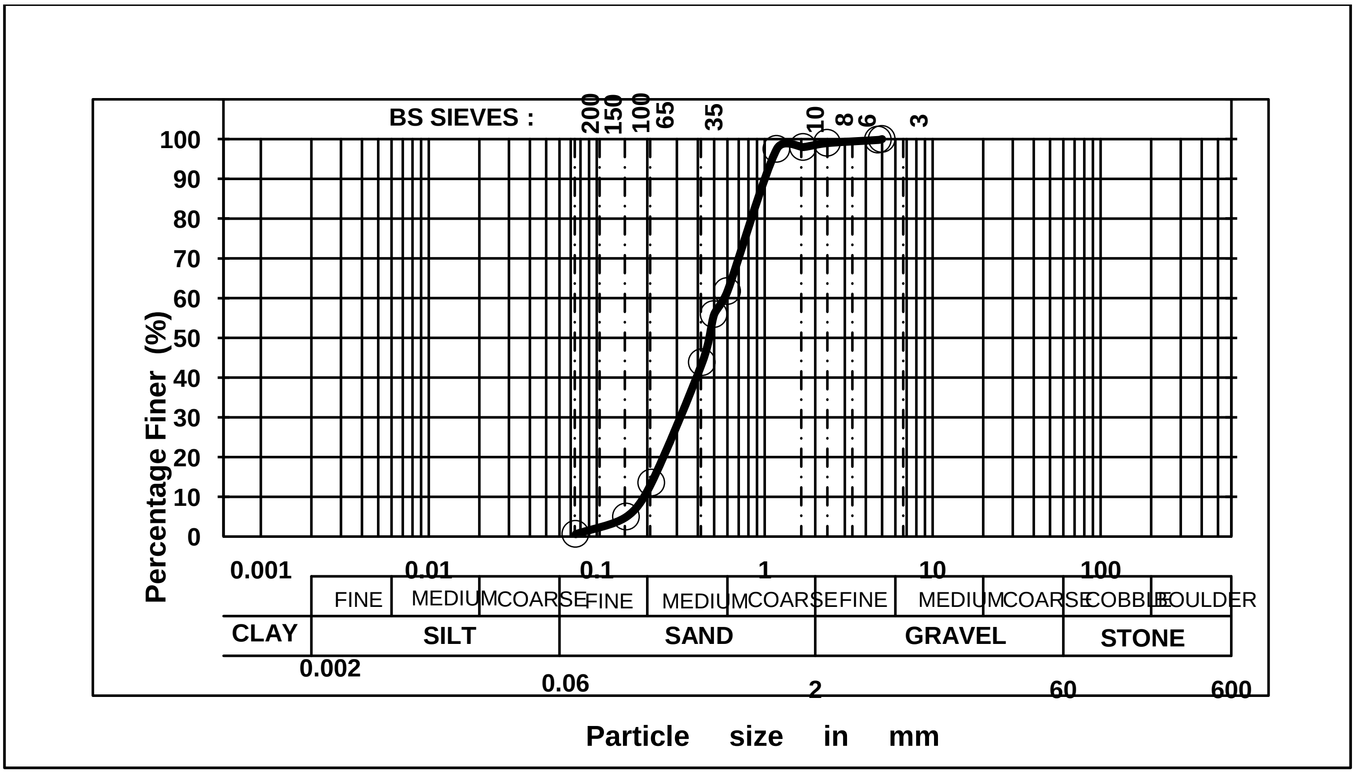


Figure 2: Particle size distribution curve for fine aggregate

3.0 Calculation of Immediate Deflection

3.1 Beam Specifications and Strength Characteristics. The space frame models were designed in accordance with the requirements of BS8110 – 1 (1997). The dimensions of the model were:

Slab: 1000mm x 1000mm x 50mm thick; Beam: 75mm x 1000mm; Column: 75mm x 75mm. Column height = 1000 mm; $f_{cu} = 7\ N/mm^2$; $f_y = 250N/mm^2$.

From design the required area of reinforcement (A_{sreq}) was 9.6 mm^2, and the provided area of reinforcement (A_{sprov}) was 2R6 bars with 56.6 mm^2 area. Deflection check according to BS 8110 -1 (1997) was satisfactory.

From Eq. (2b), and Figure 3, we have:

$$E_c = 4{,}725.64\sqrt{f_c^1} = 4{,}725.64\sqrt{7} = 12.5 * 10^3 N/mm^2$$

$$M_{RB} = 0.85\ kNm, where\ M_{RB}\ is\ moment\ of\ resistance\ of\ the\ beam$$

Since the beam- column joint is designed and detailed as fixed joint, the maximum bending moment at the mid-span is: $M = wl^2/12$

Substituting for all the relevant parameters, the estimated ultimate load $w_u = 10.2\ kN/m$ and the estimated service load $w_s = {}^{10.2}/_{1.6} = 6.38\ kN/m$.

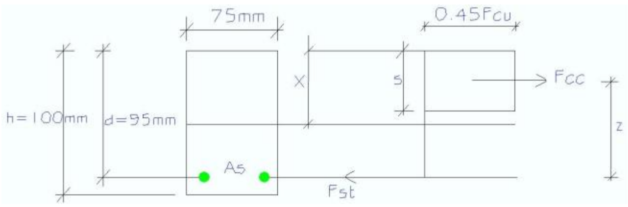


Figure 3: Stress distribution across the rectangular beam section.

3.2 Estimation of Deflection for beams Under Estimated Service Load. The beam from the square spaced framed was considered as L – beam. The beam supports un-factored dead and live loads of 0.48 kN/m and 6.38 kN/m respectively. It was built of materials with strength characteristic f_{cu} = 7 N/mm^2 for concrete, f_y = 250 N/mm^2 for steel and concrete density $\gamma = 2370\ kg/m^3$, E_c = 12.5 *10^3 N/mm^2.

3.2.1 Check if the beam has cracked at service loads. Compute I_g for the un-cracked L-section (ignore the effect of the reinforcement for simplicity):

Flange width for L-section $b_f \leq \frac{beam\ span\ length}{12} + b_w = \frac{1000}{12} + 75$ = 158 mm – according ACI 318-11 (2005).

$I_g = I_y = \sum I_{own\,axis} + \sum A\bar{y}^2 = [(164.58 + 78.13) + (204.52 + 554.69)]10^4$

$\therefore \quad I_g = 1001.97\ x\ 10^4 mm^4$

Where:

$\sum I_{own\,axis} = I_{own\,flange} + I_{own\,web}$

$I_{own\,flange} = 164.58\ x\ 10^4\ and\ I_{own\,web} = 78.13\ x\ 10^4$

i. Determine the flexural cracking moment from Eq. (5):

$M_{cr} = \frac{f_r I_g}{y_t}$

Where $f_r = 0.623\,\gamma_c\sqrt{f_c^1}\,MPa$ and $\gamma_c = 1$ for normal concrete. Using Eq. (6):

$f_r = 0.623\,\gamma_c\sqrt{f_c^1} = 0.623\ x\ 1\ x\ \sqrt{7} = 1.65\ N/mm^2$

In the positive moment region,

$M_{cr} = \frac{1.65x1001.97}{41.09} = 40.23\ N.mm = 0.04023\ x\ 10^{-3}\ kNm$

Positive moment at mid − span $= \frac{wl^2}{12}$

Dead load moment $= \frac{0.48\ x\ 1^2}{12} = 40.0\ x\ 10^{-3}\ kNm\ \ (cracked)$

$Dead\ plus\ live\ load\ loading\ \frac{(0.48 + 6.38)x\ 1^2}{12} = 0.57\ kNm\ (cracked)$

Therefore, it will be necessary to compute I_{cr} and I_e at the mid-span.

iv. Compute I_{cr} at mid-span

Taking the compression zone to be rectangular:

$I_{cr} = \sum I_{own\,axis} + \sum A\bar{y}^2$

$I_{cr} = (123.02\ x\ 10^3) + (777.48 + 4187.49)x10^3 = 5088x10^3\ mm^4$

3.2.2 Compute immediate dead-load + live load deflection. When the live load is applied to the space frame, the beam moments will increase, leading to increased flexural cracking at the mid-span. As a result, I_e will decrease.

i. Compute I_e at mid-span.

Because M_{span} = 0.57 kNm, is greater than M_{cr} = 40.23 x 10^{-6} kNm, hence the section is cracked and I_e must be determined by using Eq. (4).

$$I_e = \left(\frac{M_{cr}}{M_a}\right)^3 I_g + \left[1 - \left(\frac{M_{cr}}{M_a}\right)^3\right] I_{cr}$$

$M_a = \frac{wl^2}{12}; where\ w = unfactored\ live\ load;\ M_a = \frac{6.38\ x\ 1^2}{12} = 0.53\ kNm$

Therefore:

$$\left(\frac{M_{cr}}{M_a}\right)^3 = \left(\frac{40.23\ x\ 10^{-6}}{0.53}\right)^3 = (75.91\ x\ 10^{-6})^3 \approx 0$$

We have:

$I_e = 0\ x\ 1001.97\ x\ 10^4 + (1 - 0)x5088x10^3 = 5088\ x\ 10^3\ mm^4 = I_{cr} < I_g$

ii. Compute estimated immediate dead plus live-load deflection.

The immediate dead plus live-load deflection can be estimated at the mid-span using Eq. (11) below.

$$\Delta = \frac{5wl^4}{384E_cI_e} \qquad Eq.(11)$$

$Where, E_c = 12.5\ kN/mm^2, w_d = 0.48\ kN/mm^2, w_l = 6.38\ kN/mm^2, l = 1m, I = I_e$

Deflection due to dead load can be computed as shown below:

$$\Delta_{max} = \frac{5wl^4}{384E_cI_e} = \frac{5\ x\ 0.48\ x\ 1000^4}{384\ x\ 12.5\ x\ 5088\ x\ 10^3\ x\ 10^3} = 0.0982\ mm$$

Deflection due to estimated dead and live load at service can be computed as shown below:

$$\Delta_{max} = \frac{5wl^4}{384E_cI_e} = \frac{5\ x\ 6.38\ x\ 1000^4}{384\ x\ 12.5\ x\ 5088\ x\ 10^3\ x\ 10^3} = 1.31\ mm$$

4. RESULTS AND DISCUSSIONS

4.1 Results

The deflections measured from the experiment are presented in column 5 of Table 1. Estimated mid-span deflection can be estimated using Eq. (12). The results are presented in column 4 of Table 1.

$$\Delta_{max} = \frac{5wl^4}{384E_cI_e} = \frac{5\ x\ w\ x\ 1000^4}{384\ x\ 12.5\ x\ 5088\ x\ 10^3\ x\ 10^3} = 0.205w\ mm \qquad Eq.(12)$$

Column 1 of Table 1 shows the slab load, while column 2 shows the equivalent beam load. The estimated deflection is far lesser than the actual deflection. The ultimate beam load is 14.72 kN/m, and the service load for live load is about 66.67% of the ultimate load and this equals 9.81 kN/m. From Figure 4, the corresponding estimated deflection for service load of 9.81 kN/m is 2.01 mm, while the corresponding actual deflection is 8.14 mm. From Table 1, column 5, the actual deflection increases as the load increases. Also from Figure 4, the actual load deflection curve is not linear and deflection increases as load increases. At the beam ultimate load of 14.72 kN/m, the deflection curve flattened out and collapse of the space framed structure collapsed. Column 4 of Table 1 and Figure 4, show the estimated deflection, which increases with the load and linear throughout.

Column 6 of Table 1 shows by how much the actual deflection exceeded the estimated deflection. Between loads 0.46 kN/m and 2.30 kN/m, the estimated deflection is greater than actual deflection. As the load increases, between 0.46 kN/m and 2.30 kN/m, the gap between the estimated and the actual deflection reduces. As from 2.3 kN/m, actual deflection is greater than estimated deflection, and this deflection increases as the load increased. The percentage increase of actual deflection over the estimated one starts at 16.61% for load 2.76 kN/m and 614.05% for load 14.72 kN/m at failure. At the service load of 9.81 kN/m, the gap between the estimated and actual deflection is 305%.

From the above, I_e used in the computation of the estimated deflection is grossly inaccurate. Since the estimated deflection is lesser than actual deflection, indicates that I_e is over estimated.

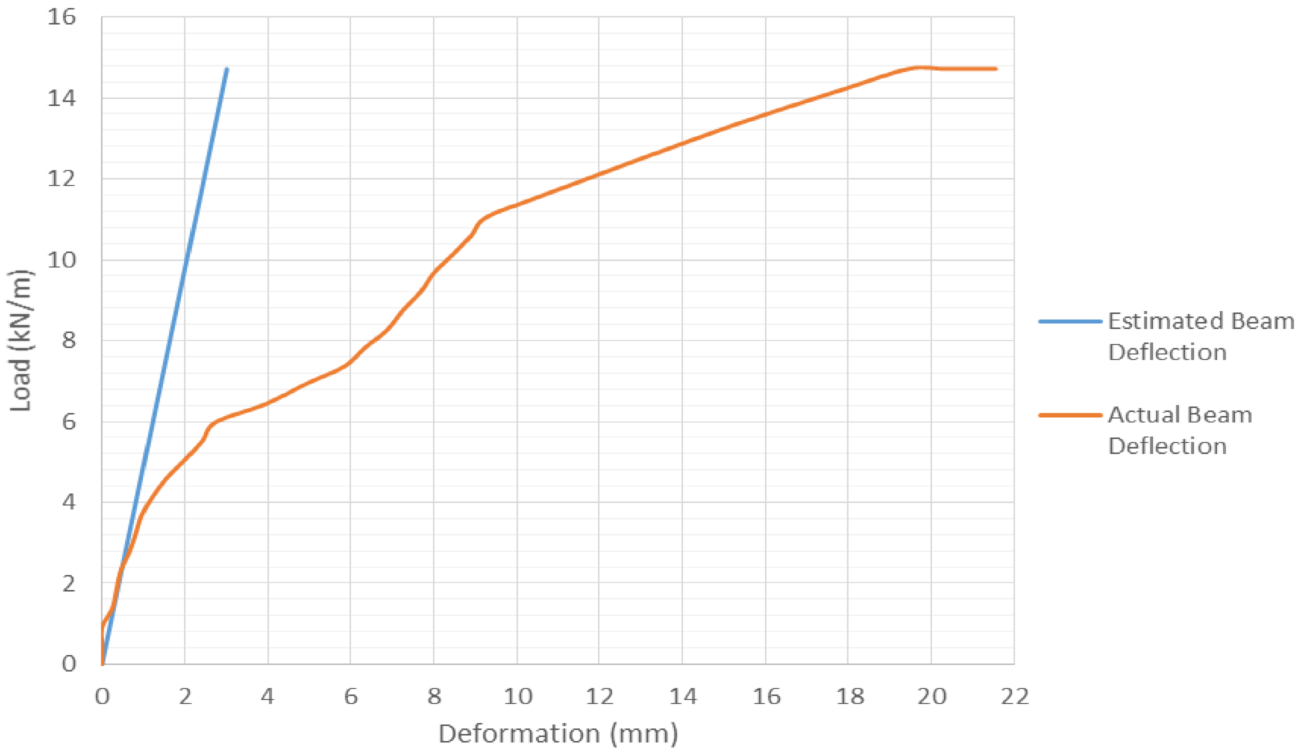


Figure 4: Estimated and Actual Load and deflection curve at the beam center

Table 1: Load, deflection

S/N 1	Slab Load (kN/m^2) 2	Load on Beam (kN/m) 3	Estimated deflection Δ_{EST} (mm) 4	Actual Deflection Δ_{ACT} (mm) 5	$\frac{\Delta_{ACT}-\Delta_{EST}}{\Delta_{EST}} x100\%$ 6
1	0	0	0	0	0
2	1.84	0.46	0.094	0.00	-100.0
3	3.68	0.92	0.189	0.00	-100.0
4	5.52	1.38	0.283	0.25	-11.66
5	7.36	1.84	0.377	0.35	-2.70
6	9.20	2.30	0.472	0.45	-4.66
7	11.04	2.76	0.566	0.66	16.61
8	12.88	3.22	0.660	0.82	24.24
9	14.72	3.68	0.754	0.96	27.32
10	16.56	4.14	0.849	1.22	43.70
11	18.40	4.60	0.943	1.56	65.43
12	20.24	5.06	1.037	2.01	93.83
13	22.08	5.52	1.132	2.42	113.78
14	23.92	5.98	1.226	2.74	123.49
15	25.76	6.44	1.320	3.97	200.76
16	27.60	6.90	1.415	4.86	243.46
17	29.44	7.36	1.509	5.84	287.01
18	31.28	7.82	1.603	6.34	295.51
19	33.12	8.28	1.697	6.89	306.01
20	34.96	8.74	1.792	7.25	304.58
21	36.8	9.20	1.886	7.68	307.21
22	38.64	9.66	1.980	7.99	303.54
23	40.48	10.12	2.075	8.45	307.23
24	42.32	10.58	2.169	8.89	309.87

Table 1: Cont'd

S/N. 1	Slab Load (kN/m^2) 2	Load on Beam (kN/m) 3	Estimated deflection (mm) Δ_{EST} 4	Actual Deflection (mm) Δ_{ACT} 5	$\frac{\Delta_{ACT}-\Delta_{EST}}{\Delta_{EST}}x100\%$ 6
25	44.16	11.04	2.263	9.25	308.75
26	46.0	11.50	2.358	10.39	340.63
27	47.84	11.96	2.452	11.59	369.00
28	49.68	12.42	2.546	12.79	402.36
29	51.52	12.88	2.640	14.01	430.68
30	53.36	13.34	2.735	15.26	457.95
31	55.2	13.8	2.829	16.61	487.13
32	57.04	14.26	2.923	18.01	516.15
33	58.88	14.72	3.018	19.45	544.47
34	58.88	14.72	3.018	20.36	574.62
35	58.88	14.72	3.018	21.55	614.05

4.2 Determination of Experimental Effective Moment of Inertia $I_{e(\exp)}$

The deflection at mid span, of the beam is calculated using equation (11) repeated as follow: $\Delta_{max}=\frac{5wl^4}{384E_cI_e}$

The experimental effective moment of inertia, $I_{e(\exp)}$ can be worked out using Equation (13) by substituting estimated deflection (Δ_{EST}) with measured deflection (Δ_{ACT}) as given by Equation (13).

$$I_{e(EXP)} = \frac{5wl^4}{384E_c\Delta_{ACT}} = 1041.67 * \frac{w}{\Delta_{ACT}} * 10^3mm^4 \quad Eq.(13)$$

Using Eq. (13), $I_{e(\exp)}$ is determine and presented in Column 4 of Table 2.

At service load of 9.81 kN/m and actual deflection 8.14 mm, the experimental effective moment of inertia, $I_{e(\exp)}$ is 1255.38 x 10^3 mm^4.

Table 2: Determination of $I_{e(\exp)}$

S/N 1	Load on Beam (kN/m) 2	Actual Deflection (mm) Δ_{ACT} 3	$I_{e(EXP)}$ (mm^4) 4	I_g (mm^4) 5	I_e (mm^4) 6	$\frac{I_e - I_{e(EXP)}}{I_{e(EXP)}}x100\%$ 7
1	0	0	0	1001.97 x 10^4	5088 x 10^3	-
2	0.46	0.00	0.00	1001.97 x 10^4	5088 x 10^3	-
3	0.92	0.00	0.00	1001.97 x 10^4	5088 x 10^3	-
4	1.38	0.25	5750.02 x 10^3	1001.97 x 10^4	5088 x 10^3	-11.51
5	1.84	0.35	5476.21 x 10^3	1001.97 x 10^4	5088 x 10^3	-7.09
6	2.3	0.45	5324.09 x 10^3	1001.97 x 10^4	5088 x 10^3	-4.43
7	2.76	0.66	4356.07 x 10^3	1001.97 x 10^4	5088 x 10^3	16.80
8	3.22	0.82	4090.46 x 10^3	1001.97 x 10^4	5088 x 10^3	24.39
9	3.68	0.96	3993.07 x 10^3	1001.97 x 10^4	5088 x 10^3	27.42
10	4.14	1.22	3534.85 x 10^3	1001.97 x 10^4	5088 x 10^3	43.94
11	4.60	1.56	3071.59 x 10^3	1001.97 x 10^4	5088 x 10^3	65.65
12	5.06	2.01	2622.31 x 10^3	1001.97 x 10^4	5088 x 10^3	94.03
13	5.52	2.42	2376.04 x 10^3	1001.97 x 10^4	5088 x 10^3	114.14
14	5.98	2.74	2273.43 x 10^3	1001.97 x 10^4	5088 x 10^3	123.80
15	6.44	3.97	1689.76 x 10^3	1001.97 x 10^4	5088 x 10^3	201.11
16	6.90	4.86	1478.91 x 10^3	1001.97 x 10^4	5088 x 10^3	244.04
17	7.36	5.84	1312.79 x 10^3	1001.97 x 10^4	5088 x 10^3	287.57
18	7.82	6.34	1284.84 x 10^3	1001.97 x 10^4	5088 x 10^3	296.00
19	8.28	6.89	1251.82 x 10^3	1001.97 x 10^4	5088 x 10^3	306.45
20	8.74	7.25	1255.75 x 10^3	1001.97 x 10^4	5088 x 10^3	305.18
21	9.20	7.68	1247.83 x 10^3	1001.97 x 10^4	5088 x 10^3	307.75
22	9.66	7.99	1259.39 x 10^3	1001.97 x 10^4	5088 x 10^3	304.00
23	10.12	8.45	1247.54 x 10^3	1001.97 x 10^4	5088 x 10^3	307.84
24	10.58	8.89	1239.69 x 10^3	1001.97 x 10^4	5088 x 10^3	310.43
25	11.04	9.25	1243.25 x 10^3	1001.97 x 10^4	5088 x 10^3	309.25
26	11.50	10.39	1152.96 x 10^3	1001.97 x 10^4	5088 x 10^3	341.30
27	11.96	11.59	1074.94 x 10^3	1001.97 x 10^4	5088 x 10^3	373.33
28	12.42	12.79	1011.54 x 10^3	1001.97 x 10^4	5088 x 10^3	403.00
29	12.88	14.01	957.65 x 10^3	1001.97 x 10^4	5088 x 10^3	431.30
30	13.34	15.26	910.61 x 10^3	1001.97 x 10^4	5088 x 10^3	458.75
31	13.8	16.61	865.45 x 10^3	1001.97 x 10^4	5088 x 10^3	487.90
32	14.26	18.01	824.78 x 10^3	1001.97 x 10^4	5088 x 10^3	516.89
33	14.72	19.45	788.35 x 10^3	1001.97 x 10^4	5088 x 10^3	545.40

4.3 Proposed Model

The model of Branson (1965) can be modified in the form of Equation (14).

$$I_e = \left(\frac{M_{cr}}{M_a}\right)^3 I_g + \left[1 - \left(\frac{M_{cr}}{M_a}\right)^3\right]\alpha I_{cr} \qquad Eq. (14)$$

Where:

α = Experimentally determined reduction factor.

$$M_a = \frac{9.81\ x\ 1^2}{12} = 0.82\ kNm$$

Since we are interested in the deflection at the service load, then I_{exp} (I_{exp} = 1255.38 x 10^3) at the service load level will be substituted in the Eq. (13) above.

$$1255.38 * 10^3 = \left(\frac{M_{cr}}{M_a}\right)^3 I_g + \left[1 - \left(\frac{M_{cr}}{M_a}\right)^3\right]\alpha I_{cr}$$

Therefore

$$\left(\frac{M_{cr}}{M_a}\right)^3 = \left(\frac{40.23\ x\ 10^{-6}}{0.82}\right)^3 = (49.06\ x\ 10^{-6})^3 \approx 0$$

We have:

$$1255.38 * 10^3 = 0\ x\ 1001.97\ x\ 10^4 + (1-0) x 5088\ x\ 10^3 \alpha = 2170.83\ x\ 10^3 \alpha$$

$$1255.38 * 10^3 = 5088\ \text{x}\ 10^3\ \alpha$$

$$\alpha = \frac{1255.38}{5088} = 0.24$$

$$\Delta_{max} = \frac{5wl^4}{384E_c\alpha I_e} = \frac{5\ x\ w\ x\ 1000^4}{384\ x\ 12.5\ x\ 0.24\ x\ 5088\ x\ 10^3\ x\ 10^3} = 0.853\ w\ mm \qquad Eq. (15)$$

Model 1: Akmaluddin and Thomas Model (2006)

The proposed effective moment of inertia (I_e), is given by the equation below:

$$I_e = I_{cre} + (I_g - I_{cre})e^{\emptyset}$$

Where:

$$I_{cre} = (0.1618 + 0.0418n\rho)\frac{bd^3}{12} = (0.1618 + 0.0418x15x0.0075)\frac{75x100^3}{12}$$

$$I_{cre} = (0.1618 + 0.0047)x625x10^4 = 1040.63x10^3$$

$$\emptyset = -\left(\frac{M_a}{M_{cr}}\right)\left(\frac{L_{cr}}{L}\right)(8.474 - 9.0607\rho + 2.842\rho^2)$$

$$L_{cr} = L\left(1 - \frac{M_{cr}}{M_a}\right)$$

$$L_{cr} = 1000\left(1 - \frac{40.23}{0.82x10^6}\right) = 999.95\ mm$$

$$\emptyset = -\left(\frac{0.82x10^6}{40.23}\right)\left(\frac{999.95}{1000}\right)(8.474 - 9.0607x0.0377 + 2.842x0.0377^2)$$

$$\emptyset = -20384.78(8.474 - 0.3416 + 0.0040) = -165.86x10^3$$

$$I_e = I_{cre} + (I_g - I_{cre})e^{\emptyset} = 1040.63x10^3 + (10019.7x10^3 - 1040.63x10^3)e^{-165.86x10^3}$$

$$I_e = 1040.63x10^3 + (8979.07x10^3)e^{-165.86x10^3} = 1040.63x10^3 + 0$$

$$I_e = 1040.63x10^3\ mm^4$$

$$\Delta_{max} = \frac{5wl^4}{384E_cI_e} = \frac{5\ x\ w\ x\ 1000^4}{384\ x\ 12.5\ x\ 1040.63\ x\ 10^3\ x\ 10^3} = 1.0\ w\ mm \qquad Eq. (16)$$

Model 2: Ammash and Muhaisin Model (2009)

$$I_e = \left(\frac{M_{cr}}{M_a}\right)^{(\varepsilon+\gamma)} * I_g + \left[\left((\varepsilon+\gamma) - \left(\frac{M_{cr}}{M_a}\right)^{(\varepsilon+\gamma)}\right)\right] I_{cr} * (\varepsilon + \beta)$$

Where

$$\beta = \frac{\rho^1}{\rho};\ \gamma = f_l - \frac{(\rho^1 + \alpha\rho)n}{\mu};\ \ \alpha = 5.26 - 0.525\left(\frac{d}{b_w}\right);\ \ \mu = \frac{I_g}{I_{cr}};\ \varepsilon = \frac{H}{L}$$

$f_l -$ factor depend on loading type such as:

1. Distributed load =1.25; 2. Two point load =1.0 and 3. Concentrated load = 0.75

$$\alpha = 5.26 - 0.525\left(\frac{95}{145}\right) = 4.92; \qquad \mu = \frac{I_g}{I_{cr}} = \frac{1001.97 * 10^4}{5088 * 10^3} = 1.96$$

$$\beta = \frac{\rho^1}{\rho} = \frac{0.01885}{0.0377} = 0.5; \qquad \varepsilon = \frac{H}{L} = \frac{100}{1000} = 0.1$$

$$\gamma = f_l - \frac{(\rho^1 + \alpha\rho)n}{\mu} = 1.25 - \frac{(0.0189 + 4.92 * 0.0377)}{1.96} = 1.15$$

$$I_e = \left(\frac{40.23}{0.82*10^6}\right)^{(0.1+1.15)} * 1001.97 * 10^4 + \left[\left((0.1 + 1.15) - \left(\frac{40.23}{0.82*10^6}\right)^{(0.1+1.15)}\right)\right] 5088 * 10^3 * (0.1 + 0.5) = 4.11 * 10^{-6} * 1001.97 * 10^4 + [(1.25) - 4.11 * 10^{-6}]5088 * 10^3 * 0.6$$

$$I_e = 4.11 * 10^{-6} * 1001.97 * 10^4 + [(1.25) - 4.11 * 10^{-6}] * 5088 * 10^3 * 0.6$$

$$I_e = 41.18 + [1.25]5088 * 10^3 * 0.6 = 3816.04 * 10^3$$

$$\Delta_{max} = \frac{5wl^4}{384EI_e} = \frac{5\ x\ w\ x\ 1000^4}{384\ x\ 12.5\ x\ 3816.04\ x\ 10^3\ x\ 10^3} = 0.273\ w\ mm \qquad Eq. (17)$$

Model 3: Bischoff's Model (2005)

The proposed effective moment of inertia (I_e), is given by the equation below:

$$\frac{1}{I_e} = \left(\frac{M_{cr}}{M_a}\right)^m \frac{1}{I_g} + \left[1 - \left(\frac{M_{cr}}{M_a}\right)^m\right]\frac{1}{I_{cr}} \qquad Eq.\ (17)$$

Substituting for the values of $I_g = 1001.97x10^4$, $I_{cr} = 5088x10^3$, $M_{cr} = 40.23$, $M_a = 0.555x10^6$ and m = 2, we have:

$$\frac{1}{I_e} = \left(\frac{40.23}{0.82 * 10^6}\right)^2 * \frac{1}{1001.97 * 10^4} + \left[1 - \left(\frac{40.23}{0.82 * 10^6}\right)^2\right] * \frac{1}{5088 * 10^3}$$

$$\frac{1}{I_e} = 0 + [1 - 0] * \frac{1}{5088 * 10^3} = \frac{1}{5088 * 10^3}$$

$$I_e = 5088 * 10^3$$

$$\Delta_{max} = \frac{5wl^4}{384EI_e} = \frac{5\ x\ w\ x\ 1000^4}{384\ x\ 12.5\ x\ 5088\ x\ 10^3\ x\ 10^3} = 0.205w\ mm \qquad Eq. (18)$$

This is the same as that of Branson's model.

4.5 Comparative Analysis of the Models

Branson's model, model 1, model 2 and the proposed model were used to estimate the deflection of the beam. The results of the estimation were presented in Table 3.

At service load of 9.81 kN/m, the estimated deflections using Branson's / Bischoff's Models, model 1, model 2 and the proposed model are 2.01 mm, 9.81 mm, 2.68 mm and 8.37 mm respectively. The percentage difference of these deflections to the actual deflection of 8.14 mm at the service load are 305%, - 17%, 204% and - 2.75% respectively. From the above, only the proposed model, did not perform badly.

Table 3: Load, deflection

		DEFLECTION (mm)				
S/N 1	Load on Beam (kN/m) 2	Branson's Deflection Model, and Model 3 3	Model 1 4	Model 2 5	Proposed model 6	Actual (mm) Δ_{ACT} 7
1	0	0	0	0	0	0
2	0.46	0.0943	0.46	0.1256	0.3924	0.00
3	0.920	0.1886	0.92	0.2512	0.7848	0.00
4	1.38	0.2829	1.38	0.3767	1.1771	0.25
5	1.84	0.3772	1.84	0.5023	1.5695	0.35
6	2.30	0.4715	2.30	0.6279	1.9619	0.45
7	2.76	0.5658	2.76	0.7535	2.3543	0.66
8	3.22	0.6601	3.22	0.8791	2.7467	0.82
9	3.68	0.7544	3.68	1.0046	3.1390	0.96
10	4.14	0.8487	4.14	1.1302	3.5314	1.22
11	4.60	0.9430	4.60	1.2558	3.9238	1.56
12	5.06	1.0373	5.06	1.3814	4.3162	2.01
13	5.52	1.1316	5.52	1.5070	4.7086	2.42
14	5.98	1.2259	5.98	1.6325	5.1009	2.74
15	6.44	1.3202	6.44	1.7581	5.4933	3.97
16	6.90	1.4145	6.90	1.8837	5.8857	4.86
17	7.36	1.5088	7.36	2.0093	6.2781	5.84
18	7.82	1.6031	7.82	2.1349	6.6705	6.34
19	8.28	1.6974	8.28	2.2604	7.0628	6.89
20	8.74	1.7917	8.74	2.3860	7.4552	7.25
21	9.20	1.8860	9.20	2.5116	7.8476	7.68
22	9.66	1.9803	9.66	2.6372	8.2340	7.99
23	10.12	2.0746	10.12	2.7628	8.6324	8.45
24	10.58	2.1689	10.58	2.8883	9.0247	8.89
25	11.04	2.2632	11.04	3.0139	9.4171	9.25
26	11.50	2.3575	11.50	3.1395	9.8095	10.39
27	11.96	2.4518	11.96	3.2651	10.2019	11.59
28	12.42	2.5461	12.42	3.3907	10.5943	12.79
29	12.88	2.6404	12.88	3.5162	10.9866	14.01
30	13.34	2.7347	13.34	3.6418	11.3790	15.26
31	13.8	2.8290	13.8	3.7674	11.7714	16.61
32	14.26	2.9233	14.26	3.8930	12.1638	18.01
33	14.72	3.0176	14.72	4.0186	12.5562	19.45

5.0 CONCLUSION AND RECOMMENDATIONS

5.1 Conclusions

Branson's / Bischoff's Model and model 2 grossly under estimated the deflection by 305% and 2043% respectively, while model 1 grossly overestimated the deflection by 17%, and the proposed model in this study overestimated the deflection by just 2.75%. Therefore all the existing models performed badly. From the above, Branson's / Bischoff's Model and model 2 overestimated the effective moment of inertia, while model l under estimated the effective moment of inertia.

The beam was satisfactory using the span/effective depth ratio. However, the actual deflection at service load for this experimental work was 8.14 mm which exceeded the maximum permissible computed deflections (ACI 318, 2005) of L/480, which equals 2.08 mm. Therefore, non-structural elements, such as partition walls, supported by such beams are likely to be damaged by large deflections, and therefore the beam is not satisfactory in deflection.

From the above, it is most likely that structures which deflection criteria were based on span/effective depth ratio is likely to fail in deflection, as there is the possibility of the occurrence of damage in terms of cracks of non-structural elements such as partition walls.

5.2 Recommendation

Based on the above conclusions, the following recommendations are made:

i. Research should be conducted on the effect of concrete grade on the effective moment of inertia using locally available materials.

ii. Effects of reinforcement percentage on the effective moment of inertia using locally available materials should be investigated.

iii. The span/effective depth ratio alone should not be used in checking for deflection, rather this should be complemented by actual deflection calculation.

DECLARATIONS

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Author's contributions

The author of this research paper have directly participated in the planning, execution, or analysis of this study and have read and approved the final version submitted.

Conflict of interest statement

I hereby states that, there is no conflict of interest whatsoever with any third party.

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