

A Comparative Analysis Method of Seismic Quasi-Static on Stability of Earth Dams

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ABSTRACT: Given the need for in habiting and storing of surface water, to provide water for drinking, needed water for agriculture and industry, power generation, flood controlling and river flooding, and also the first step in the development and utilization of water resources is creating high dams. Technical and economic considerations at the design of high dams has shown that in many cases dams with clay core is preferred over other types of dams and is a good choice for the final design. Considering that one of the main affecting issues in the analysis and design of earth dams of the earthquake-prone areas, is the evaluation of seismic stability of these dams, in this study, it was attempted that by using the Geo-Studio software, the different methods of the quasi-static seismic analysis and with the approach of equilibrium methods will be compared with each other. And accordingly the best option in order to design these dams will be offered

Keywords: Analysis of Quasi-Static, Limit Equilibrium, Seismic Stability, Stability of Earth Dams

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INTRODUCTION

From the past, dams' construction for adjusting and storing of water was common, however, due to limited facilities, and lack of knowledge of soil mechanic sand hydraulics laws, the height of dams and earth end version dams don't exceed of certain value. Although in terms of the extent and length of the dam, this restriction was not present. But now with advances in soil mechanics and technology facilities development and more accurate studies, the possibility of development and construction of earth dam with considerable height have been provided.

Among the benefits of earth dams, the use of natural materials and relatively simple procedure and of course, lower construction cost, can be mentioned. The most common type of these dams is earth dam with a clay core, which central clay core and is responsible for sealing function and maintaining the water behind the dam. Due to the coherence properties of clay and it's abundance on nature in the most cases, make the core of earth dams of clay.

Considering that earth dam are including important geotechnical structures that failure on them can lead to compensatory damages, Thus, it is necessary to apply all necessary control-and sensitivities on designing them. One of these cases is seismic stability controlling of dam that in earthquake-prone areas has tremendous importance. Considering that a set of affecting parameters and the relationships between them on seismic stability analysis of earth dams is varied, Therefore, the study of the stability of earth dams always is considered as one of the most complex issues in the field of geotechnical structures. Considering the extent of these dams application and also their locating in areas with high seismicity, estimating the stability of these dams will have crucial role in dam engineering.

Given the above, one of the major problems on the design and construction earth dams, is the using seismic stability of soils slopes issue in the earth dams. Forth is purpose, different methods seismic stability analysis of soil slopes is used that of the most widely used methods of seismic stability analysis of soils slopes, the dynamic analysis techniques and quasi-static can be referred. On the method of dynamic analysis must by using suitable accelerograms and finite element analysis, the slop safety factor will be determined. And while that on the quasi-static methods of analysis based on the seismic coefficient equation and limit equilibrium methods will be carried perform. The main advantage of equilibrium methods can refer to the ease of use and acceptable accuracy of these methods that these cases hassled to the widespread use of these methods.

During the past three decades approximately one dozen methods of slices have been developed (Fellenius, 1936; Bishop, 1955; Duncan and Wright, 2005; Janbu, 1954, 1973; Spencer, 1967; Morgenstern, 1963, 1965, 1967; Zhu et al., 2005). They differ in these methods in (i) the statics employed in deriving the factor of safety equation and (ii) the assumption used to render the problem determinate. One of the differences in the slope stability analysis method is failure surface type that is considered. A number of methods are considered circular and a number, as well as the ability to match any type of failure surface can be disruptive.

One of the most important researches in the past few decades is Fredlund and Krahn (1977) the paper compares six methods of slices commonly used for slope stability analysis. All equations are extended to the case of a composite failure surface and also consider partial submergence, line loadings, and earthquake loadings and presented a new derivation for the Morgenstern-Price's method (Fredlund and Krahn, 1997).

Seed (1979) by examining a number of earthquakes determine the affecting parameters on seismic soil structure behavior introduced seismic analysis of quasi-static on the earthen structures. In this article, he, provide coefficients as maximum accelerated deflator of earthquake (Seed, 1979). Seed et al. (1975) examined the San Fernando dam based on seismic analysis of dynamical and obtained results is compared with the actual results and assessed the behavior of this dam during the occurred earthquake properly.

The proposed derivation is more consistent with that used for the other methods of analysis but utilizes the elements of statics and the assumption proposed by Morgenstern and Price (1965). The Newton-Raphson numerical technique is not used to compute the factor of safety and λ . (Seed, 1975). Griffiths and Lane (1999), in this paper a comparison between limit equilibrium method of slices and strength reduction has been done. Hang and Huang (2005), in this paper a soil slope with 10 m in height was analyzed by limit equilibrium methods and strength reduction.

In this research it has been tried, that seismic analysis by using Geo-Studio software based on different methods the quasi-static Limit Equilibrium and, for modeled dam will be taken and, obtained results for evaluation of the mentioned methods are studied and compared. Given the earth dams stability issue, in practice, is related to the stability of used soil slopes on the earth dam, consequently, on this part, equilibrium methods of slope earth analysis is presented and studied.

Limit Equilibrium Methods of Slices

In the limit equilibrium method of slices we must satisfy critical slip surface, at first. The Factor of Safety (FS) is defined as the ratio of resisting to driving forces on a potential sliding surface. The slope is considered safe only if the calculated safety factor clearly exceeds unity. Most problems in slope stability are statically indeterminate, and as a result, some simplifying

assumptions are made in order to determine a unique factor of safety.

Due to the differences in assumptions, various methods have been developed. Among the most popular methods are procedures proposed by Fellenius, Bishop, Janbu, Spencer and Morgenstern-Price's methods referred to before. Some of these methods satisfy only overall moment, like the Ordinary and simplified Bishop Methods and are applicable to a circular slip surface, while Janbu's method satisfies only force equilibrium and is applicable to any shape. Spencer and Morgenstern-Price's methods, however, satisfies both moment and force equilibrium and it is applicable to failure surfaces of any shape. It is considered as one of the rigorous and accurate methods for solving stability problems. Table 1 presents a summary of static equilibrium conditions in limit equilibrium methods of slices considered in this study.

A typical two dimensional slope has been shown in Figure 1. In this Figure resistant and deriving forces have been shown as a sample. In limit equilibrium methods of slices we must divide the upper soil profile in a number slices. By these considerations we can explain limit equilibrium methods of slices as follow.

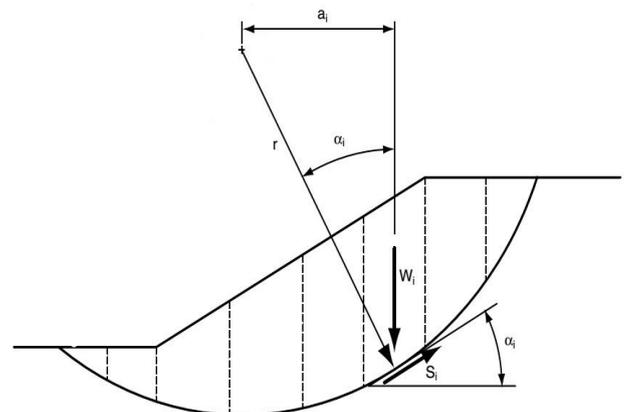


Figure 1. A typical slope by some slices

Table 1. Static equilibrium conditions in limit equilibrium methods of slices

Method	Assumption	Failure surface	Equilibrium equation satisfied	Solution by
Swedish method (Fellenius, 1927)	Resultant of interslice force is zero; $J_s = 0$	Circular	Moment	Calculator
Bishop's simplified method (Bishop, 1955)	E_j and E_{j+1} are collinear; $X_j - X_{j+1} = 0$, $J_s = 0$	Circular	Moment	Calculator
Bishop's method (Bishop, 1955)	E_j and E_{j+1} are collinear; $J_s = 0$	Circular	Moment	Calculator/computer
Morgenstern and Price (1965)	Relationship between E and X of the form $X = \lambda f(x)E$; $f(x)$ is a function ≈ 1 , λ is a scale factor, $J_s = 0$	Any shape	All	Computer
Spencer (1967)	Interslice forces are parallel; $J_s = 0$	Any shape	All	Computer
Bell's method (Bell, 1968)	Assumed normal stress distribution along failure surface; $J_s = 0$	Any shape	All	Computer
Janbu (1973)	$X_j - X_{j+1}$ replaced by a correction factor, f_{α} ; $J_s = 0$	Noncircular	Horizontal forces	Calculator
Sarma (1975)	Assumed distribution of vertical interslice forces; $J_s = 0$	Any shape	All	Computer

Ordinary method of slices

For the Ordinary method of slices (Fellenius, 1936), which is considered the simplest method of slices, the factor of safety is directly obtained. The method assumes that the inter-slice forces are parallel to the base of each slice, thus they can be neglected and the factor of safety is given as follows:

$$FS = \frac{\sum [c \cdot \Delta l_i + (w_i \cos \alpha_i - u_i \Delta l_i \cos^2 \alpha_i) \cdot \tan \varphi]}{\sum w_i \sin \alpha_i} \quad (1)$$

Where:

$$w_i = \gamma \cdot b_i \cdot h_i$$

c = Cohesion

Δl_i = Area of the base of the slice for a slice of

unit thickness

α_i = Angle of the base of slice

w_i = Weight of slice

γ = Unit weight of soil

U = Pore water pressure

b_i = The width of the slice

h_i = The height of the slice at the centreline

φ = Internal friction angle

FS = Factor of safety

Simplified Bishop's method

In Bishop's method (Bishop, 1995; Duncan and Wright, 2005) the factor of safety is determined by trial and errors, using an iterative process, since the factor of safety (FS) appears in both sides of Eq. (2). The inter-slice shear forces are neglected, and only the normal forces are used to define the inter-slice forces. The factor of safety is given as follows:

$$FS = \frac{\sum_{i=1}^n \left[\frac{c \cdot \Delta l_i \cdot \cos \alpha_i + (w_i - u_i \Delta l_i \cos \alpha_i) \tan \varphi}{\cos \alpha_i + (\sin \alpha_i \cdot \tan \varphi) / FS} \right]}{\sum_{i=1}^n w_i \cdot \sin \alpha_i} \quad (2)$$

Input parameters were defined as upper.

On Bishop Method to determine the safety factor by using seismic analysis of quasi-static the following equation is used.

$$FS = \frac{\sum_{i=1}^n \left[\frac{c \cdot \Delta l_i \cdot \cos \alpha_i + w_i \cdot \tan \varphi}{\cos \alpha_i + (\sin \alpha_i \cdot \tan \varphi) / FS} \right]}{\sum_{i=1}^n w_i \cdot \sin \alpha_i + \left(\sum_{i=1}^n k_h \cdot w_i \cdot d_i \right) / r}$$

k_h = the horizontal coefficient of earthquake

Simplified Janbu's method

Similarly, for Janbu's method (Duncan and Wright, 2005; Janbu, 1954; Janbu 1973) the factor of safety is determined also by an iterative procedure through varying the effective normal stress on the failure surface. The inter slice shear forces are ignored and the normal forces are

derived from the summation of vertical forces. The resulting factor of safety is given below:

$$FS = f_0 \cdot \left(\frac{\sum_{i=1}^n \left[\frac{c \cdot \Delta l_i \cdot \cos \alpha_i + w_i \cdot \tan \varphi}{\cos^2 \alpha_i + (\sin \alpha_i \cdot \cos \alpha_i \cdot \tan \varphi) / FS} \right]}{\sum_{i=1}^n w_i \cdot \tan \alpha_i} \right) \quad (3)$$

Where:

$$\text{For } c, \varphi > 0 \quad f_0 = 1 + 0.5 \left[\frac{D}{L} - 1.4 \left(\frac{D}{L} \right)^2 \right] \quad (4)$$

Where:

f_0 = Correction factors

L = The length joining the left and right exit points

D = The maximum thickness of the failure zone with reference to this line another procedure for f_0 determination.

Spencer's method

In Spencer's method (Spencer, 1967), the effect of inter-slice forces is included and both moment and force equilibrium are explicitly satisfied. This eventually will lead to an accurate calculation of the factor of safety. The factor of safety is determined through an iterative procedure, slice by slice, by varying FS and θ until force and moment equilibrium are satisfied.

The equation for force equilibrium can be written as

$$\sum_{i=1}^n Q_i = 0 \quad (5)$$

Where Q_i is the resultant of the inter slice forces, and for moment equilibrium, moments can be summed about any arbitrary point. Taking moments about the origin ($x=0, y=0$) of a Cartesian coordinate system, the equation for moment equilibrium is expressed as

$$\sum_{i=1}^n Q_i \cdot (x_{b_i} \cdot \sin \theta - y_{b_i} \cdot \cos \theta) = 0 \quad (6)$$

Where X_b is the x (horizontal) coordinate of the centre of the base of the slice and Y_b is the y (vertical) coordinate of the point on the line of action of the force, Q_i , directly above the centre of the base of the slice. Q_i is determined by following equation:

$$Q_i = \frac{w_i \cdot \sin \alpha_i - c \cdot \Delta l_i + w_i \cdot \cos \alpha_i \cdot \tan \varphi / FS}{\cos(\alpha_i - \theta) + \sin(\alpha_i - \theta) \cdot \tan \varphi / FS} \quad (7)$$

Where:

θ = Inter-slice force inclination

Morgenstern-Price's method

The Morgenstern and Price's procedure (Morgenstern, 1963; Morgenstern and Price, 1965 and 1967) assumes that the shear forces between slices are related to the normal forces as:

$$X = \lambda \cdot f(x) \cdot E \quad (8)$$

Where X and E are the vertical and horizontal forces between slices, λ is an unknown scaling factor that is solved for as part of the unknowns, and $f(x)$ is an assumed function that has prescribed values at each slice boundary. In the Morgenstern-Price's method, factor of safety is determined by following equation (Zhu et al., 2005).

$$FS = \frac{\sum_{i=1}^{n-1} \left(R_i \prod_{j=1}^{n-1} \psi_j \right) + R_n}{\sum_{i=1}^{n-1} \left(T_i \prod_{j=1}^{n-1} \psi_j \right) + T_n} \quad (9)$$

Where R_i is the sum of the shear resistances contributed by all the forces acting on the slices except the normal shear inter-slice forces, and T_i is the sum of the components of these forces tending to cause instability? Where:

$$\psi_i = \left[(\sin \alpha_{i+1} - \lambda \cdot f_i \cdot \cos \alpha_{i+1}) \cdot \tan \phi + (\cos \alpha_{i+1} + \lambda \cdot f_i \cdot \sin \alpha_{i+1}) \cdot FS \right] / \phi_i \quad (10)$$

$$\phi_i = (\sin \alpha_i - \lambda \cdot f_i \cdot \cos \alpha_i) \cdot \tan \phi + (\cos \alpha_i + \lambda \cdot f_i \cdot \sin \alpha_i) \cdot FS \quad (11)$$

Differences in the assumptions of each method lead to differences in the related solutions to each method. In this study, it has been tried that the levels of difference on the resulted safety factor of the all above methods using the solved example will be determined.

Seismic analysis of quasi - static

Given that Seismic Analysis is one kind of remarkable and important analysis on stability of earth dams reviewing shall be done for earth structures that are constructed in seismic areas, this type of analysis should be conducted. Given that during the earthquake occurring, the behavior of structures is dynamic behavior, therefore, to obtain reliable answers dynamic seismic analysis should be conducted. But since for dynamic seismic analysis performing ,significantly experimental and computational cost is essential, the simplified and equivalent analysis can be used that assume earth structures as the static behavior. The determined results by using this method, with simply unacceptable enjoys acceptable accuracy.

On the analysis of quasi-static a coefficient as Seismic coefficient as horizontal or vertical on the considered slope is entered. In other words, on the seismic analysis of quasi-static, the entered force by earthquake as the weight of each piece coefficient on the center of each piece is entered. A typical described case is shown in the Figures (2,3 and 4).

Procedure of analysis

One of the most important steps in slope stability analysis by limit equilibrium methods of slices is critical slip determination. In this paper for critical slip surface optimization, we use optimization algorithm by SLOPE/W software. For this purpose, a field must be considering for center and another field for radius. In Figure 4 a flowchart has been represented for Schematic representation of methodology used.

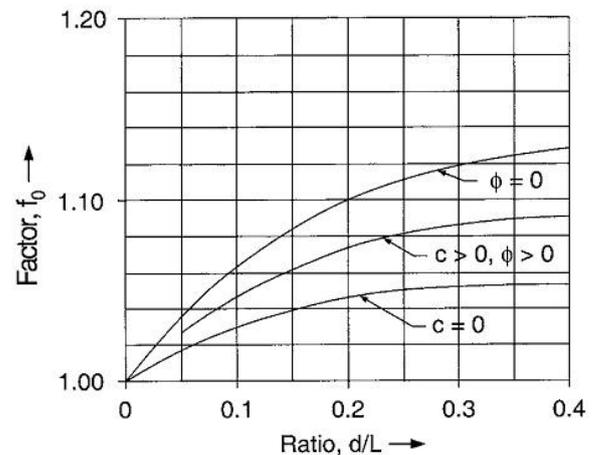
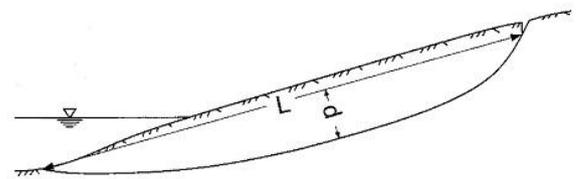


Figure 2. Determine the f_0

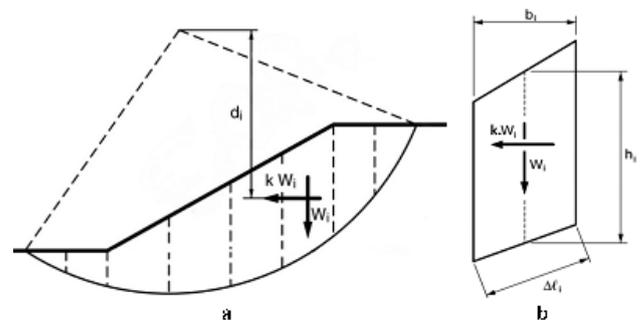


Figure 3. (a). Slope with weight and seismic forces, (b). Dimensions for an individual slice

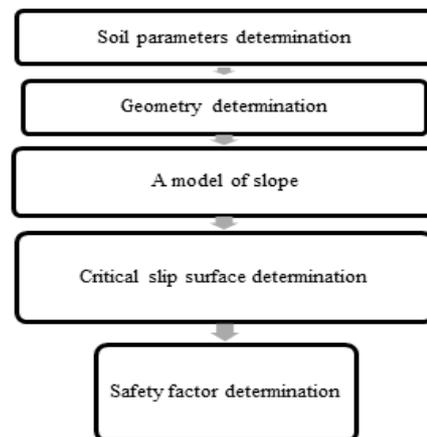


Figure 4. Schematic representation of methodology

A Case Study

To examine the accuracy of the methods in determining the safety factor of earth dams, an example with arbitrary parameter values is demonstrated. In the solved example, San Fernando earth dam as an example is analysed and evaluated that this dam due to the earthquake occurring in some parts of the dam is failure. In Figure 5 a Cross-section of the earth dam is provided that used parameters in this analysis, in the Table 2 are shown in (Seed, 1979; Seed et al., 1975).

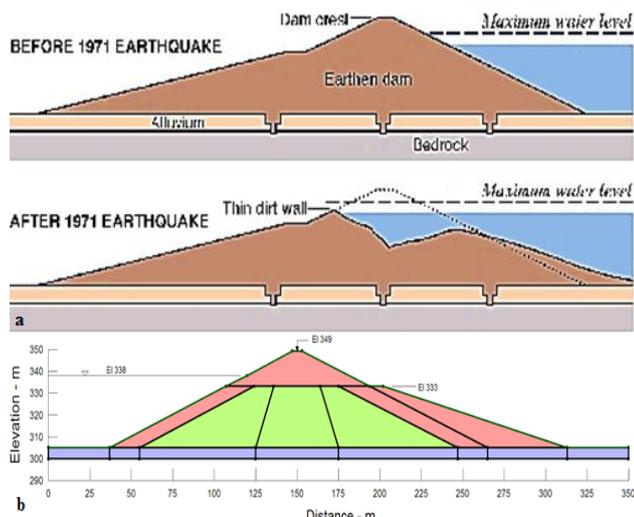


Figure 5. (a). Overall view of San Fernando's dam (before and after the earthquake) (Seed, 1975); (b). Modeled earth dam based on San Fernando dam (Seed et al., 1975)

Table 2. Used parameters in the case study

Soil type	C (kPa)	Φ (degree)	γ (kN/m ³)
Soil of earth dam's core	0.0	34.0	18.0
Soil of earth dam's body	5.0	34.0	18.0
Soil of earth dam's basement	5.0	36.0	20.0

RESULTS

After modelling of the considered dam, in the SLOPE / W software (Figure 5) and performed stability analysis of the dam, the obtained results in Tables 3 and 4 are provided. In Figure 5 and 6 are shown the critical failure surface for upstream and downstream of dam. In Table 3 the resulting safety factor for upstream and downstream of the considered dam without applying for an earthquake and also in Table 4 similar results, for imposed horizontal seismic coefficient state (on earthquake applying) are presented.

Table 3. Comparison of determined safety factors in the various methods without applying the earthquake effect

Method	Ordinary	Bishop	Simplified Janbu	Spencer	Morgenstern-Price
Safety factor of upstream	1.677	1.860	1.809	1.823	1.807
Safety factor of downstream	2.150	2.352	2.211	2.262	2.241

Table 4. Comparison of determined safety factors in the various methods by applying the earthquake effect (quasi-static analysis)

Method	Ordinary	Bishop	Simplified Janbu	Spencer	Morgenstern-Price
Safety factor of upstream	0.733	0.840	0.821	0.829	0.824
Safety factor of downstream	1.128	1.422	1.139	1.214	1.193

According to Table 3, it can be concluded that the smallest amount of safety factor is determined in the case of no applying of the caused force by earthquake for modelled dams upstream and downstream using Ordinary method. And the highest safety factor is calculated based on Bishop. According to Table 4 Ordinary and Bishop Methods respectively provide the lowest and highest levels of safety factor for upstream and downstream. As the presented results in Tables 3 and 4 can be seen, the results of the methods of Simplified Janbu and Spencer and Morgenstern-Price are pretty close together.

There have been efforts in the following field by using performed analysis, the inter-piece vertical and shear forces for different methods are compared to each other. As mentioned in the previous sections, the difference in the determined results using different methods is due to each of these methods assumptions variation. In this section we try to fully numerically evaluate these differences. And examine the effect of these assumptions on determined safety factor obviously. The presented results are related to the analysis of quasi-static form deled earth dams upstream.

In Figure 10a comparison of the shear force on the leg of each piece and the applied driving force at the foot of the various components on the critical failure surface schematically is provided for all methods. Given these results it is clear that mobilized Shear force on thereof pieces to leg of the slope initially was low and by moving

to the breasts of slope this force is gradually rising. After arriving to the mid-slope the mobilized shear force reaches its maximum value and then began to decline and at the top of the slope, this decrease is linear.

According to the presented results in Table 4, determined safety factor for Quasi-static analysis of dams upstream are less than unity and this indicates that the amount of the driving forces on the critical failure surface are more than resisting forces and as you can see in the following Figure, it is shown correctly. In these Figures, driving forces in blue and resisting forces is shown in red that indicate the dams' safety factor is smaller than the unity.

Given that determined safety factor on the critical failure surface is equal to the ratio of resisting forces to the driving forces, therefore different values for different methods are determined based on their assumptions about the forces. Since the determination of these forces on the considered calculations is so important, therefore, in Figure 11 the determined resisting force by using various methods is compared. On Figure 12 also the driving forces have been investigated and compared. As can be seen the determined resisting method using the Morgenstern-Price and Ordinary methods are minimal.

As it can be seen the defined driving forces using the Ordinary highest and Bishop Method are lowest respectively. On Figure 13 the caused horizontal force by the earthquake that is applied in different parts of the up

streams lobe of the embankment dam will be displayed. As this Figure shows, the earth quake force at the beginning is low and at slope middle reaches its maximum value. Note that the applied seismic force to the analysis

of quasi-static is equal to percentage of the weight of each piece, on the below gradients, by moving toward the middle of slop ,the components weight increased and as a result the earth quake will be added swell.

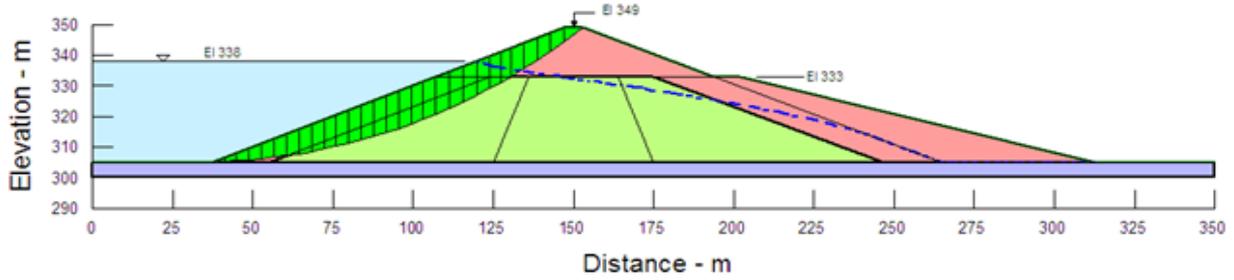


Figure 6. Critical failure surface of earth dams upstream

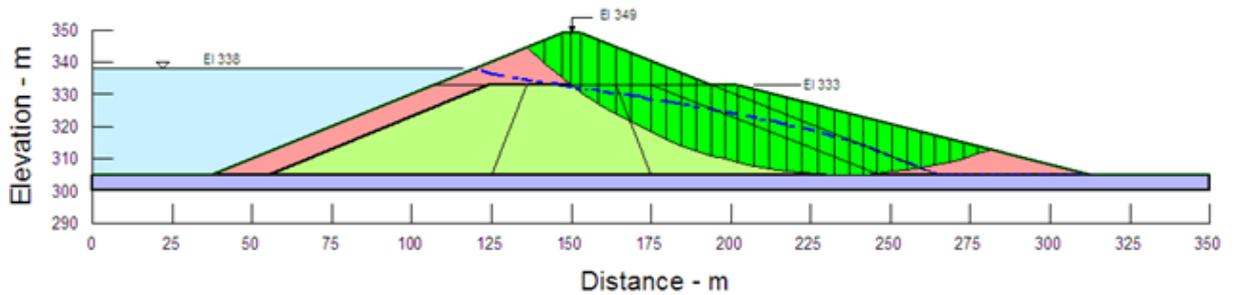


Figure 7. Critical failure surface of earth dams downstream

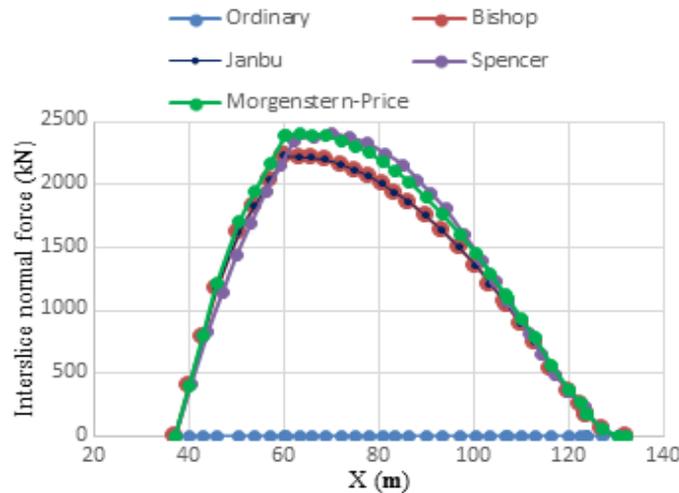


Figure 8. Comparison of the vertical inter-piece force to different methods of Limit Equilibrium

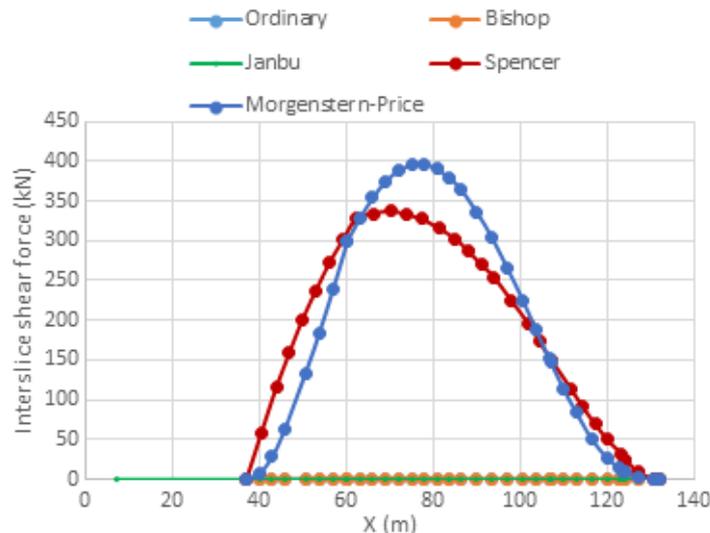


Figure 9. Comparison of the shear inter-piece force to different methods of Limit Equilibrium

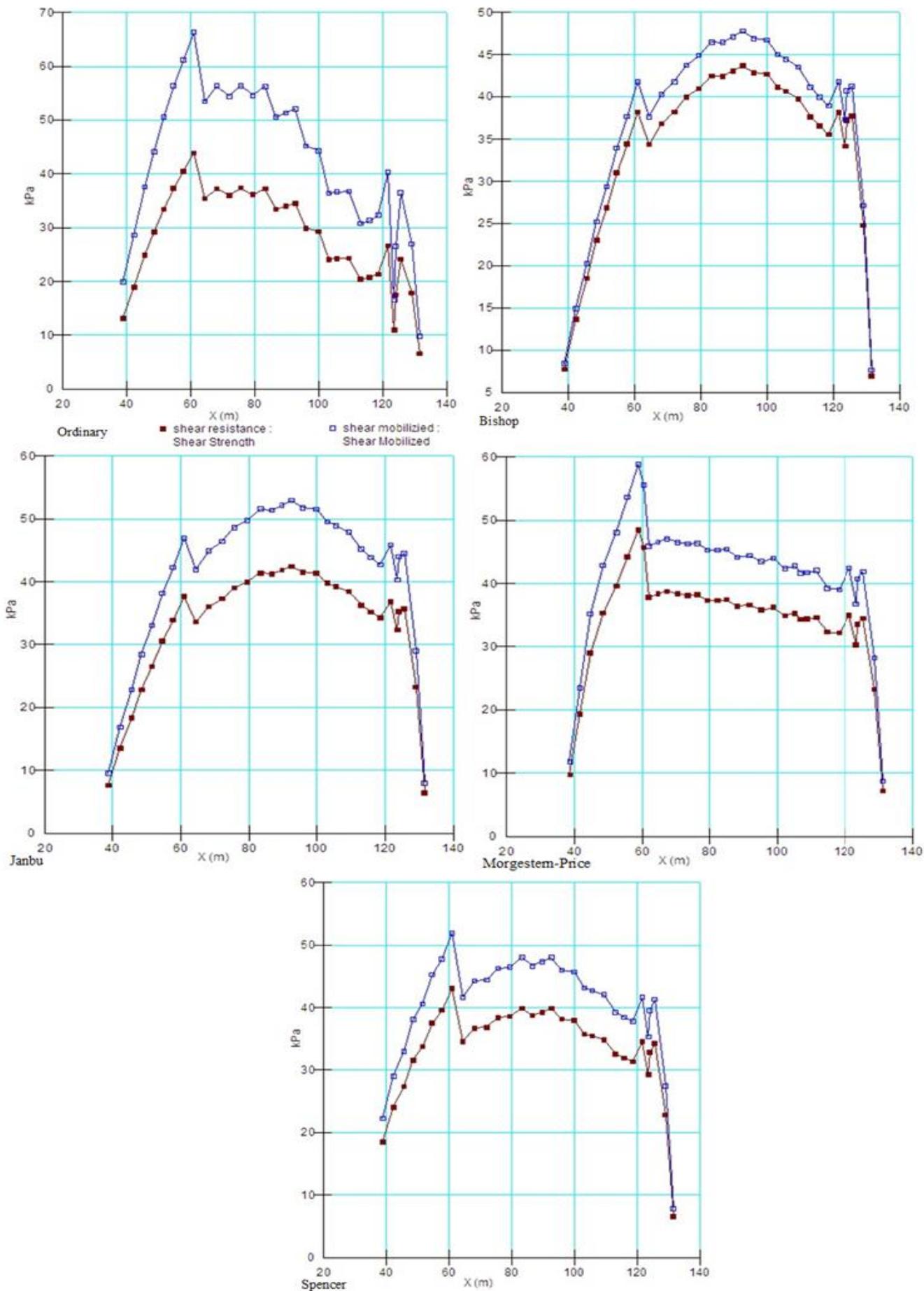


Figure 10. Comparison between the driving and the resistance force at the critical failure on earth dams for different ways of Limit Equilibrium Piece

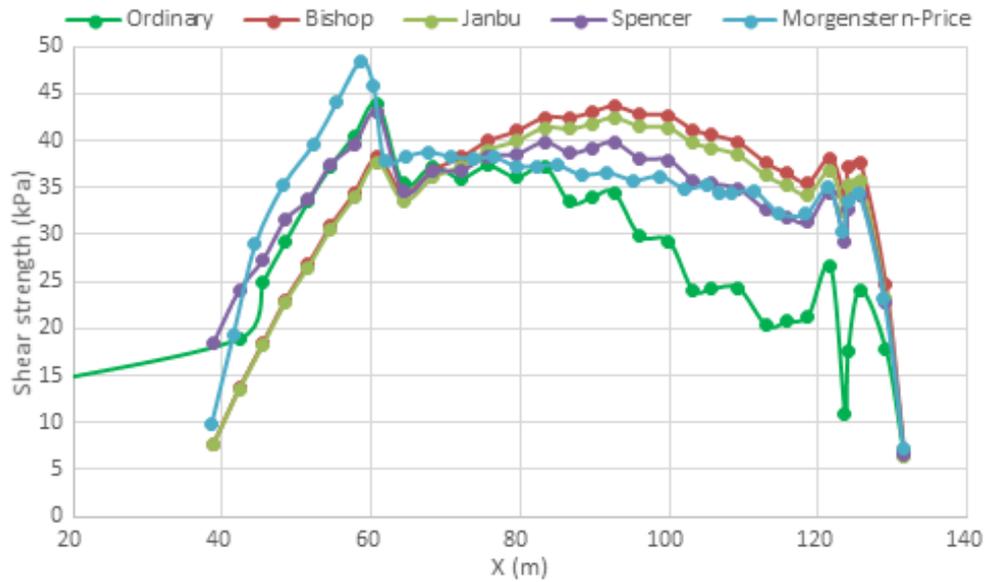


Figure 11. Comparisons between the determined resisting forces on failure surface.

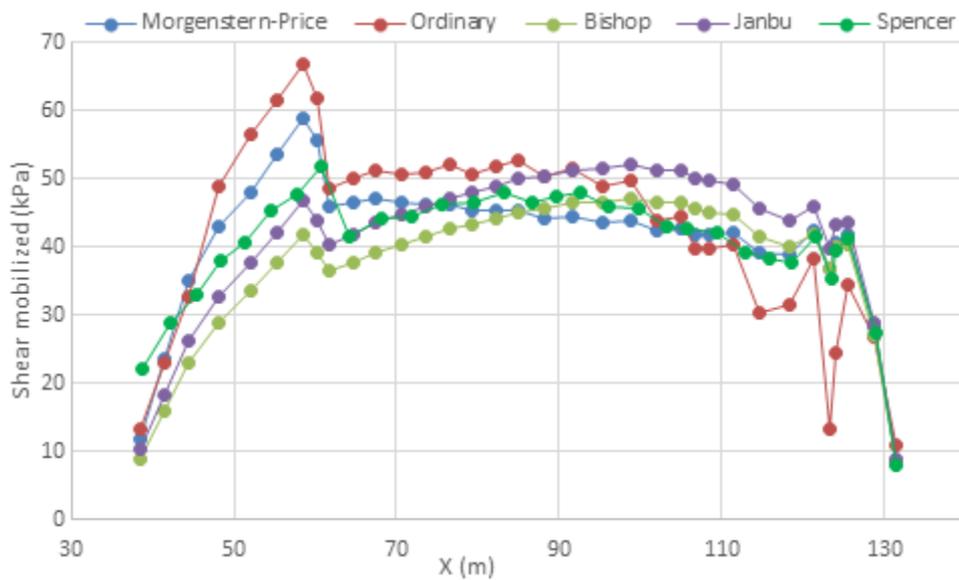


Figure 12. Comparison of the driving force in the pieces leg by various methods

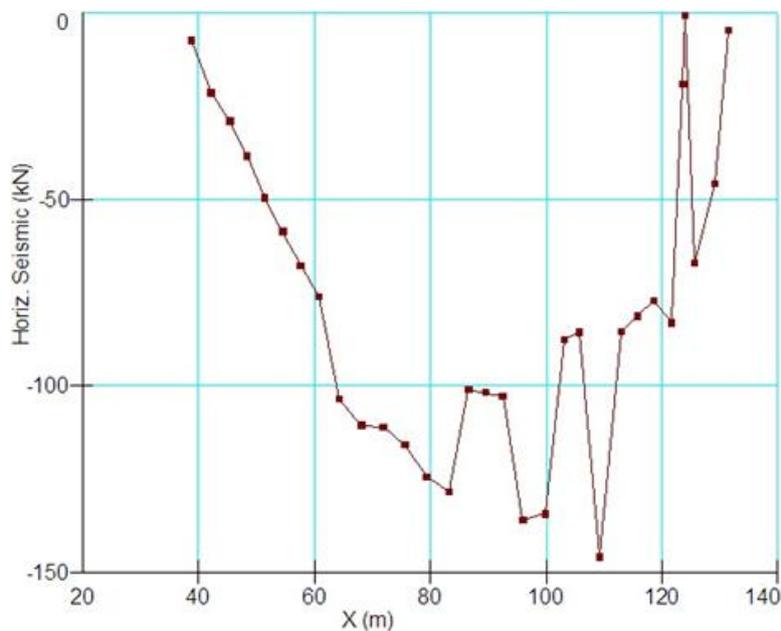


Figure 13. Forces due to applied earthquake to the various components

DISCUSSION AND CONCLUSION

Given the importance of the stability of earth dam and the existence of various ways of Limit Equilibrium, it is necessary that answers and assumptions of different methods properly will be compared to each other. In this research, it has been effort that different ways of balance so will be compared to each other and the impact of different assumptions on the results properly will be considered. According to Table 3 it can be concluded that the smallest amount of safety factor in the case of not applying the caused force by the earthquake for upstream and downstream of the modelled dam is determined using Ordinary method and most safety factors are calculated based on Bishop. According to Table 4 Bishop and Ordinary methods, respectively the minimum and maximum values of safety factor for upstream and downstream have been offered. As the presented results in Tables 3 and 4 it can be seen, the result of Simplified Janbu, Spencer and Morgenstern-Price methods, are pretty close to each other. Considering the Morgenstern-Price method and Spencer than other previous methods are complementary techniques and on these methods specifically, the Morgenstern-Price method are more considered logical assumptions, It seems that, these methods results that are within the minimum and maximum range of , the resulting solutions are more reliable. By considering the determined safety factor on Critical failure surface is equal to the ratio of resisting forces to the driving forces, consequently, different methods, based on the original assumptions assigned to these forces different values. Accordingly, a comparison - between the designated driving and also was done using any method and as you have seen designated resisting forces, by using Morgenstern-Price the high and Ordinary methods are the lowest. On the other hand, as observed the designated driving force using the Ordinary high and Bishop are the lowest.

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