Effect of a High Resolution Finite Volume Scheme with Unstructured Voronoi Mesh for Dam Break Simulation

Hamidreza Jalalpour* and Seydeh Mona Tabandeh

Department of Civil Engineering, Science and Research Branch, Islamic Azad University, Tehran, Iran

*Corresponding author’s E-mail: hamidrejazalalpour@yahoo.com

ABSTRACT: In this paper, a high resolution finite volume method (FVM) is developed in order to discretization of multidimensional mathematical equations of Dam Break Phenomena using an unstructured Voronoi mesh grid for evaluating the effect of this type of mesh. Accordingly, the robust local Lax–Friedrichs (LLxF) scheme was used for the calculating of the numerical flux at cells interfaces. The model named FVDBC was run under the asymmetry partial and circular dam break conditions and then verified by comparing the model outputs with the documented results presented in the literature. A statistical analysis using SPSS statistical are performed to confirm the verification of the developed method. In addition, statistical observations indicated a good conformity between those output and documented results, the FVDBC could be considered as a reliable method for dealing with shallow water (SW) and shock problems, especially those having discontinuities.

Keywords: Dam break, Finite volume method, High resolution, Local Lax–Friedrich scheme, Voronoi Mesh.

INTRODUCTION

Although safety criteria are considered through the design, construction and operation of dams, they may break under unpredictable events such as huge floods, war or earthquake. One of the most important applications of dam break analysis is preparing emergency action plan (EAP). An EAP is a formal plan that determines the potential emergency conditions at a dam and prescribes the procedures that need to be followed in order to minimize the property damage and loss of life. These entail numerical and laboratory investigations of dam break flows and their potential damage. The shallow water equations (SWEs) are conventionally used to describe the unsteady open channel flow such as dam break. These equations are named as Saint Venant equations for one-dimensional (1D) problem and also include the continuity and momentum equations for two dimensional (2D) studies. Many researchers studied the dam break problem, such as Toro (2001), Wu et al. (1999), Wang and Shen (1999), Mohapatra et al. (1999), Zoppou and Roberts (2000), Wang et al. (2000), Wang and Liu (2000), Venutelli (2003), Ponce et al. (2003), Zhou et al. (2004), Liang et al. (2004), Quecedo et al. (2005), Begnudelli and Sanders (2006), Loukili and Soula’mani (2007), D’iaz et al. (2008) and Aliparast (2009), especially using computational fluid dynamic (CFD) methods. Recently, Xia et al. developed a 2D morphodynamic model for predicting dam break flows over a mobile bed (Xia et al., 2010). Erpicum et al. presented a 2D finite volume (FV) multiblock flow solver, which was able to deal with the natural topography variation (Erpicum et al., 2010).

Baghlani utilized a combination of the robust and effective flux-difference splitting (FDS) and flux-vector splitting (FVS) methods to simulate dambreak problems based on FVM on a Cartesian grid. The method combined the effectiveness and robustness of the FDS and FVS methods to precisely estimate the numerical flux at each cell interface (Baghlani 2011). Zhang and Wu developed a hydrodynamic and sediment transport model for dam break flows. The 2D SWEs were solved based on the FVM with an unstructured quadtree mesh grid (Zhang and Wu, 2011). Singh et al. developed a 2D numerical model to solve the SWEs for the simulation of dambreak problems (Singh et al., 2011).

Chang et al. (2011) proposed a meshless numerical model to investigate the shallow water (SW) dam break in 1D open channel. A numerical model was used to solve the SWEs based on smoothed particle hydrodynamics (SPH). The concept of slice water particles was adapted in the SPH–SWE formulation. Shakibaenia and Jin (2011) developed a new mesh-free particle model based on the weakly compressible MPS (WC-MPS) formulation for modeling the dam break problem over a mobile bed. Sarveram and Shamsai (2012) investigated the dam break problem in converge and diverge rectangular channels in the unsteady stance using Saint Venant equations and a quasi-Lagrangian method. This paper attempts to present a novel development for 2D dam break problems. A high-resolution FVM is employed to solve the SWEs on unstructured Voronoi mesh. The local Lax–Friedrichs (LLxF) scheme is used for the estimation of fluxes at cells and the numerical approximation of hyperbolic conservation laws.
MATERIAL AND METHODS

In this article, the studied domain was discretized using unstructured Voronoi meshes with MATLAB software. In the Voronoi mesh, the chosen point has lower distance in the devoted domain rather than other points. If one point has the same distance from several domains, it will be divided between domains. Indeed, these points create Voronoi cell boundaries. Consequently, internal sections of the Voronoi mesh consist of nodes belonging to one domain and boundaries include nodes that belong to several domains.

Figure 1. A cell of Voronoi mesh in domain \( \Omega \)

Governing equation

The continuity and momentum equations of the SW can be written in different forms depending upon the requirements of the numerical solution of governing equations. The 2D SWEs with source terms are given in the vector form considering a rigid bed channel as follows:

\[
\begin{align*}
U_x + F_x + G_y &= S \\
U_y &= \left[ \begin{array}{c} h \\ hu \\ hv \end{array} \right] = \left[ \begin{array}{c} q_x \\ q_y \\ q_z \end{array} \right] \\
F_x &= \left[ \begin{array}{c} \frac{q_x^2}{h} + \frac{1}{2} gh^2 \\ \frac{q_x q_y}{h} \\ \frac{q_x q_z}{h} \end{array} \right] \\
G_y &= \left[ \begin{array}{c} \frac{q_y^2}{h} + \frac{1}{2} gh^2 \\ \frac{q_x q_y}{h} \\ \frac{q_y q_z}{h} \end{array} \right] \\
S &= \left[ \begin{array}{c} gh(S_{0x} - S_{fx}) \\ gh(S_{0y} - S_{fy}) \end{array} \right]
\end{align*}
\]

Where: \( U \) is the vector of conserved variables; \( F \) and \( G \) are the flux vector functions; \( S \) is the vector of source terms; \( u \) and \( v \) are velocity components in the \( x \) and \( y \) directions, respectively. Also \( h \) is water depth and \((S_{0x}, S_{0y})\), are bed slopes in the \( x \) and \( y \) directions; \((S_{fx}, S_{fy})\) are friction slopes in the \( x \) and \( y \) directions respectively. The friction slopes are estimated by using the Manning formulas as given in Equation (6):

\[
S_{fx} = \frac{n^2 u v}{h^{1.33}} , \quad S_{fy} = \frac{n^2 u v}{h^{1.33}}
\]

Where \( n \) = Manning’s roughness coefficient. In the case of dam-break flow, the influence of bottom roughness prevails over the turbulent shear stress between cells. Therefore the effective stress terms were neglected in the computation.

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Governing equation discretization

In order to solve discrete equations, the parameter in each node was computed considering its discrete equation and newest adjacent nodes’ values. Solution procedure can be expressed as follows:

1. (1) Assuming an initial value in each node as an initial condition.
2. (2) Calculating the value in a node considering its discrete equation.
3. (3) Performing previous step for all nodes over the studied domain, one cycle is performed by repetition this step.
4. (4) Verifying the convergence clause. If this clause is satisfied, the computing will end otherwise the computations will be repeated from the second step.

In this section, Discretization of the equation 1 is performed applying FVM with unstructured Voronoi mesh in domain \( \Omega \). It should be mentioned that F face vertexes nomination direction is counter clockwise from b to a centring o (Figure 2).

Figure 2. The 2D schematic Voronoi mesh cell used for describing the discretization of the governing equations.

\[
\frac{\partial}{\partial \varphi} \int_{\varphi} \vec{u} \cdot d \varphi + \int_{\varphi} \vec{H}(u) d \varphi = \int_{\varphi} S(u) d \varphi
\]

Where \( H(u) \) is input and output flux to voronoi cell that contains \( G(u) \) and \( F(u) \) functions in \( x \) and \( y \) directions. By implementing divergence theorem:

\[
\int_{\varphi} \vec{H}(u) d \varphi = \oint_{\Omega} n \cdot H(u) d \Omega
\]

By applying the Voronoi mesh, (8) is yielded to (9) for an investigated control volume; however, Figure 2 illustrates the 2D Voronoi mesh grid used for describing these equations as follows:

\[
\frac{\partial}{\partial \varphi} \int_{\varphi} u d \varphi + \int_{\varphi} \Delta H(u) d \varphi = \int_{\varphi} S(u) d \varphi
\]
\displaystyle \frac{\partial u}{\partial t} A + \sum_f \left( \mathbf{H}(u) \cdot \mathbf{n} \right)_f = S(u) \quad (10)

Where, 
\( A \) is the investigated Voronoi cell area, \( A_f \) is the Voronoi cells side’s area and \( \mathbf{n} \) is the normal vector. Then,
\displaystyle \frac{\partial u}{\partial t} = \frac{1}{2} \sum_f \left( \mathbf{H}(u) \cdot \mathbf{n} \right)_f + S(u) \quad (11)
\displaystyle \mathbf{A}_f = (A, \mathbf{e}_f) \quad (12)
\displaystyle \mathbf{H}(u) = H(u)_x \mathbf{e}_x + H(u)_y \mathbf{e}_y \quad (13)

Where \( \mathbf{e}_x \) the unit outward normal vector in each Voronoi cell and \( \mathbf{e}_y \) is the unit tangent vector in each Voronoi cell.
\[ \sum \mathbf{A}_f \mathbf{H}(u) = A_f H(u) \quad (14) \]
\[ \frac{\partial u}{\partial t} = \frac{1}{2} \sum_f H(u)_x A_f + S(u) \quad (15) \]
\[ \int_t^{t+\Delta t} d_t = \frac{1}{2} \sum_f H(u)_x A_f d_t + \int_t^{t+\Delta t} S(u) d_t \quad (16) \]
\[ U^{n+1} - U^n = \frac{1}{2} \sum_f H(u)_x A_f \Delta t + S(u) \Delta t \quad (17) \]
\[ H(u)_x = n_1, F(u), n_2, G(u) \quad (18) \]
\[ n_1 = \varepsilon_x = \frac{(x_b - x_p)}{\Delta_x} \quad (19) \]
\[ n_2 = \varepsilon_y = \frac{(y_b - y_p)}{\Delta_y} \quad (20) \]
\[ \Delta_x = \sqrt{(x_b - x_p)^2 + (y_b - y_p)^2} \quad (21) \]

Where: \( n_1 \) is the investigated side cosine in each Voronoi cell; \( n_2 \) is the investigated side sine in each Voronoi cell; \( x_p, y_p \) are the evaluated cell center coordinates; \( x_b, y_b \) are the neighbor cells centers coordinate; \( \Delta_x \) is the distance between the center of investigated Voronoi cell and its neighbors. 

Discrete equation can be written as equation (22):
\[ U^{n+1} = U^n - \frac{\Delta t}{A} \sum[n_1 F(u)_x A_f + n_2 G(u)_x A_f] + S(u) \Delta t \quad (22) \]

Where:
\( F(u)_x, G(u)_x \) are Voronoi cell normal flux vectors and \( \Delta t \) is the interval that computed from courant number clause.

The Local Lax-Friedrichs high order scheme in Voronoi mesh

With expanding equation (22):
\[ U^{n+1}_p = U^n_p - \frac{\Delta t}{A} \sum (G_f(v)_{n_{about}} A_f + n_{2f}) - \sum (G_f(v)_{n_{about}} A_f + n_{2f}) + S(u) \Delta t \quad (23) \]

We can compute intercell flux by implementing this method as given in Equations (24) to (29):
\[ F_f(v)_{\mathbf{n}_{\text{bout}}} = \frac{f(u^n_p + f(u^n_b))}{2} \left( \frac{v^n_b + v^n_p}{2} \right) \left( u^n_b - u^n_p \right) \quad (24) \]
\[ F_f(u)_{\mathbf{n}_{\text{bout}}} = \frac{f(u^n_p + f(u^n_b))}{2} \left( \frac{v^n_b + v^n_p}{2} \right) \left( u^n_p - u^n_b \right) \quad (25) \]
\[ \lambda = \bar{u} + \sqrt{gh} \quad (26) \]
\[ \frac{g(v^n_p + g(v^n_b))}{2} G_f(v)_{\mathbf{n}_{\text{bout}}} = \frac{1}{2} \left( \frac{v^n_b + v^n_p}{2} \right) \left( v^n_b - v^n_p \right) \quad (27) \]
\[ G_f(v)_{\mathbf{n}_{\text{bout}}} = \frac{g(v^n_p + g(v^n_b))}{2} \left( u^n_p - u^n_b \right) \quad (28) \]
\[ \lambda = \bar{v} + \sqrt{gh} \quad (29) \]

After computing intercell flux by utilizing Local Lax-Friedrichs scheme, the equation can be solved and the final result can be calculated after each time step. The \( \Delta t \) value should compute by using courant Friedrichs lewy (CFL) after each time step as follow.
\[ \Delta t = CFL \cdot min \left( \frac{\Delta x}{\max|A_f|}, \frac{\Delta t}{\max|x|} \right) \quad (30) \]

The CFL should range over \([0, 1]\) for achieving to the stability (\(0 < CFL < 1\)).

\[ \lambda_1 = \bar{u} + \sqrt{gh} \quad (31) \]
\[ \lambda_2 = \bar{v} + \sqrt{gh} \quad (32) \]

RESULTS AND DISCUSSION

Validation: The numerical model is validated using the example of dam break for which data is available.

Two dimensional anti-symmetric dam break test: Many researcher, have implemented this test. As shown as Figure 3 the channel that is located in horizontal bed has 200 meters length, 200 meters width it contains an anti-symmetric cut with 75 meters width. The channel and dam domain are assumed frictionless. The upstream water depth is proposed 10 meters and the downstream water depth is assumed 5 meters. In this test we proposed 40*40 node points and the suggested total time for comparing values is 7.2 second. Meshing zone is shown in Figure 4.

![Figure 3, the plan of 2D studied domain for verification](image3)

![Figure 4, A sample of Voronoi mesh in this model](image4)
Figure 5. Anti-symmetric dam break test in a frictionless, horizontal domain after 7.2 second based on presented model (with Voronoi mesh)

Figure 6. Anti-symmetric dam break test in a frictionless, horizontal domain after 7.2 second based on presented model (with rectangular mesh)

Figure 7. Anti-symmetric dam break test in a frictionless, horizontal domain after 7.2 second

For a closer look, a diagram of water depth Variation for different type of meshes and references is plotted.

Results of statistical t test analysis to evaluate the effect of various meshes are shown in this table. t test is one of the common and accurate tests that used to compare results. The computed P-Values explain the accuracy of model. If the P-Values closer to 1 more certitude of model achieved.

Table 1. P-VALUE resulted from statistical t test

<table>
<thead>
<tr>
<th>Reference</th>
<th>Mesh type</th>
<th>P-Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wang &amp; Lu (2000)</td>
<td>Voronoi</td>
<td>0.973</td>
</tr>
<tr>
<td></td>
<td>Rectangular</td>
<td>0.967</td>
</tr>
<tr>
<td>Yuling &amp; Wenli (2005)</td>
<td>Voronoi</td>
<td>0.959</td>
</tr>
<tr>
<td></td>
<td>Rectangular</td>
<td>0.948</td>
</tr>
</tbody>
</table>

Two dimensional dam break circular test: In this test we have considered a frictionless and horizontal rectangular domain that has 200 meters length and 200 meters width. The initial conditions consisted of two states separated by a circular discontinuity. The computational grid consists of 40 *40 cells and the radius of the circle r = 50 meters and it is centered at x = 100 meters. The water depth outside the circle is 1 meter deep and inside the circle is 10 meters deep. The water depth has shown after 2 second. This result demonstrates that the model is able to simulate complicated geometries.

Figures 9 and 10 illustrate output results of FVDBC code including the water surface formation and water depth contours, respectively.
For comparing results a diagram of water depth variation for different type of meshes and references is plotted. We can see that the type of Voronoi mesh is more coincident than the rectangular one. Results of statistical t test analysis to evaluate the effect of various meshes are shown in this table.

<table>
<thead>
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</thead>
<tbody>
<tr>
<td>Wang &amp; Lu (2000)</td>
<td>Voronoi</td>
<td>0.996</td>
</tr>
<tr>
<td></td>
<td>Rectangular</td>
<td>0.974</td>
</tr>
</tbody>
</table>

CONCLUSION

In the present research, a novel and friendly user code was written. It is observed that the LLxF scheme along with the FVM method by using unstructured Voronoi grid is a suitable combination in order to simulate 2D dam break problems. Through the obtained results, there was not, significant numerical dispersion problem or nonphysical alternation. This consequence was predicted because of combination of FVM and Voronoi mesh. The computed P-Values are between 0.95-1 that demonstrates high ability of presented mathematical-numerical model in the evaluation of dam break. FVM have many merits rather than other numerical methods such as, suitable compatibility with studied domain especially in dam break problem, easy prescription for shallow water equation discretization.

REFERENCES


