

Runoff Estimation Based on Stochastic Optimization Method

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ABSTRACT: Despite widespread use of Curve Number (CN) as main parameter in SCS equations for estimating runoff of basins with limited data, its capability in appropriate estimation of runoff has always been controversial. In this study, ability of CN method has been studied in estimating runoff through the application of a stochastic optimization method and within the framework of SCS equations. In stochastic method, sensitivity of statistical distributions, number of statistical samples and range of CN changes on accuracy of calculations have been studied through the application of Monte Carlo Simulation. The used stochastic method is based on the rainfall and runoff information. For this purpose, rainfall and runoff modeling are first selected and probable distribution functions, governing these events, are specified. Then, rainfall and runoff modeling are produced in different sample sizes within random modeling. Produced samples of rainfall and runoff are evaluated based on the specific criteria and specific runoff modeling is selected. The selective runoffs and number of experimental curves are used in SCS equations, rainfall modeling is calculated and their probable functions are determined. Based on Kolmogorov-Smirnov Test, optimal curve number (CN_{OPT}) is set between produced distribution functions and initial probability distribution function governing rainfall events. Also, average and mean curve number (CN_{AVG} , CN_{MEG}) has been studied. Statistics of basins representing Kasilian and Emameh (Northern Iran) has been used with the aim of studying efficiency of stochastic method. In Kasilian Basin, the runoffs, calculated through the use of CN_{OPT} , enjoyed the minimum error than observational runoffs. This emphasizes desirable capability of CN method in estimating runoff of the basin but stochastic method was unenforceable in Emameh Basin due to the shortage of rainfall and runoff events.

Keywords: Curve Number, Monte Carlo Simulation, SCS Method, Statistical Distribution Function, Stochastic Method.

ORIGINAL ARTICLE

INTRODUCTION

Recognizing specifications of hydrological and meteorological phenomena, such as intensity and duration of rainfall, time and spatial distribution of rainfall and runoff, is of paramount importance in studying flood volume and severity and its effect on soil and plant and protective instruments (Alizadeh 2010).

In many of the estimated cases, the runoff rate of basins is necessary for designing hydraulic structures, measuring sediment and characteristics of basin in case of inexistence of enough information. For materializing the mentioned objective, the U.S. Soil Conservation Service (SCS), affiliated to the United States Department of Agriculture (USDA), presented the Curve Number (CN) for estimating and evaluating runoff rate produced from rainfall related to the direct runoff volume and physical specifications of the basin.

The CN changes in each rainfall event and has not a constant rate. But the important point is that relatively long rainfall modeling can be used for attaining a specific and constant rate of the curve number which is regarded as CN of the basic index. In these cases, since soil infiltration rate is reached to the final capacity and rainfall is enough for the continuation of runoff, CN near to constant number. For determining curve number of the

basic parameter, it seems necessary to use selected modeling of the rainfall and runoff. The selected rainfall modeling should enjoy enough time continuity, aimed at creating necessary condition for reaching to the final capacity of infiltration and producing runoff.

Yu (1998) states basic hypothesis for obtaining SCS runoff relation as follows: equaling ratio of actual maintenance to the potential maintenance with the ratio of actual runoff to the runoff potential. Although this hypothesis has not been clarified theoretically and empirical, Yu clarified the initial relation, which SCS relation obtained from it, through the use of water final infiltration rate in soil and also clarification of rainfall time distribution function and infiltration spatial distribution function as exponential. Regarding to the data selection and application of the selected events, different methods have been presented in determining CN both in statistical and hydrological terms. (Hawkins 1990; Hawkins et al. 1984; Hawkins 1993; Rallison et al. 1979; Rallison and Miller 1981)

Rainfall and runoff frequency distribution has always been taken into consideration. Asymptote Method was introduced by Hawkins in 1993, based on which, the secondary relation is created in basic between CN and rainfall height (P). In most cases, CN will reach to a constant value if "P" is increased. This constant value is

set by the tangent which is drawn in extreme on curve of CN relation and P after application of frequency matching. Hawkins (1990) divided basins into three groups based on type of their behaviors against rainfall modeling as follows: quiet, Standard and Intense (Hawkins 1990).

Hjelmfelt (1980) showed that using greater modeling of rainfall for the provision and application of rainfall frequency distribution and runoff will produce better result in determining curve number as well.

Bonta (1997) presented an applied stochastic method for determining basin CN through the use of Hjelmfelt method, in which, rainfall empirical distributions is used of watershed basin. Ansarifard (1992) considers SCS empirical method as an appropriate method for determining CN of basins representing Emameh and Kasilian in Iran. Moreover, it seems that this method is one of the best methods in determining flood and design for other basins. Nahvi (1992) determined CN of Emameh and Kasilian Basin through the use of physiographic data of basin and percent of land area with vegetation.

Nahvi (1992) has calibrated SCS equation for these two basins through the application of the least-squares test, based on which, the difference square of the observed and estimated runoffs was minimized.

Kerrou et al. (2010) assumed a geo-statistical model of the exploitation rates based on a multi-linear regression model combining incomplete direct data and exhaustive secondary information by Monte Carlo simulation. The impacts of the uncertainty on the spatial distribution of the pumping rates on seawater intrusion were evaluated using a 3-D density-dependent groundwater model.

Svensson et al. (2013) presented a method that allows all the input variables such as rainfall to take on values across the full range of their individual distributions. These values were then brought together in all possible combinations as input to an event-based rainfall-runoff model in a Monte Carlo simulation approach. Frequency analysis was applied to the annual maximum peak flows and flow volumes.

In this paper, statistical distribution, governing rainfall and runoff events, has been taken into consideration and its efficiency will be compared in estimating CN and evaluation with common methods. Also, other methods will be used for determining CN like percent of land area, asymptote method, mean and average method and experimental procedure for estimating performance of stochastic method with relation to the other methods. In stochastic method, sensitivity of number of statistical samples, and sensitivity of changes of CN will be studied on accuracy of calculations through use of Monte Carlo Simulation.

MATERIALS AND METHODS

In this case study, Kasilian and Emameh basin in northern Iran, which have more statistical background than other basins, have been selected. Emameh area is about 37.2 square kilometer and located between heights (from 1900 to 3868m) in northern part of Jajroud river basin, regarded as one of its basins, above Latyan Dam. The basin also is situated between north latitudes of 35°

5'20" and 51°38'30" and east longitudes of 51°21'30" and 51° 38'30". The average height of the basin is measured approx. 2,650m (Water Resources Research Organization, 1994).

The basin representing Kasilian is the second basin which has been established in Iran. This basin represents vast areas of mountainous and forested parts of north Alborz. The mentioned basin is located in the vicinity of Tehran-Gorgan Railway Road and 18th km of east Pol-eSefid City. This basin is situated between north latitudes of 35° 58' 30" and 36° 7' 0" and east longitude of 53° 10' 30" and 53° 18' 0".

Emameh includes three meteorological stations and Kasilian includes three meteorological stations. The rainfall and runoff statistics, measured in meteorological and hydrometric stations of Emameh and Kasilian basins, has been received from Iran Water Resources Research Organization. Minimum 30-year statistics is at hand in two mentioned basins and there is not any limitation in this respect, but the selection procedure of rainfall and runoff modeling is too important. In such modeling large enough data with the aim of showing physiographic specifications of the basin is required.

Although, large floods have high return period this cannot be used for determining CN of the basin at required time range. Determining curve number of watershed basin index is of paramount importance in hydrological, hydraulic, irrigation and drainage projects. Stochastic method for determination of CN with the help of relation presented by SCS and using rainfall and runoff frequency distributions (thanks to the existing statistics in the field of watershed basin) is a giant stride towards improving estimation of CN. Monte Carlo Simulation is used for optimizing number of experimental curves within the framework of a number of different statistical fields. The basic relationship for the method, presented by the U.S. Soil Conservation Service (SCS), is explained as follows:

$$Q = \frac{(P - I_a)^2}{P - I_a - S} \quad (1)$$

"P" represents rainfall height (mm), "Q" represents runoff height (mm), "S" represents the factor related to the maintenance of water surface (mm), "I_a" represents initial absorption of water (mm).

Since this relationship is considered as (I_a = 0.2S), equation (1) is turned into the following equation:

$$Q = \frac{(P - I_a)^2}{P + 0.8S}, \quad P > 0.2S$$

$$Q = 0, \quad P \leq 0.2S \quad (2)$$

The relationship between CN and S is explained as follows:

$$CN = \frac{25400}{254 + S} \quad (3)$$

If, in Equation (2), CN amount is substituted from Equation (3) instead of S and if it is solved for "P", the following equation will be obtained:

$$P = B + \frac{\{Q[Q + \frac{1016}{CN}(100 - CN)]\}^{1/2}}{2} \quad (4)$$

Wherein:

$$B = \frac{Q}{2} + \frac{50.8}{CN}(100 - CN) \quad (5)$$

In this study, equations (4) and (5) have been used for calculating rainfall through using selective runoffs (produced) and number of experimental curves. Then, rainfall modeling are calculated through probability distributions functions and basin optimal CN will be estimated through comparing them with the probability distribution function of rainfall modeling according to the method which is explained. Since rainfall modeling is calculated through produced runoffs, it is necessary to study relationship between rainfall and runoff probability distribution functions. The general relationship of stochastic between P, Q can be explained as follows:

$$Q = f(P) \quad (6)$$

In the above relationship, "Q" values have been stated randomly. In other words, if $f(P)$ is assumed rainfall frequency distribution values as independent data, the values "Q" will be estimated randomly. On the other hand, the above relationship can be used to solve rainfall data as well.

$$P = f^{-1}\left(\frac{P-0.2S}{P+0.8S}\right) \quad (7)$$

$$P = f^{-1}(Q) \quad (8)$$

According to the studies carried out by Benjamin & Cornell (1970), the probability that random variable of "Q_r" turns smaller or equal to the actual (experimental) "Q_a" ($Q_r \leq Q_a$) is equaled to the probability that random variable of "P_r" turns smaller or equal to the actual "P_a".

$$F_Q(Q) = \text{Prob}(Q_r \leq Q_a) \quad (9)$$

$$F_P(P) = \text{Prob}(P_r \leq P_a) \quad (10)$$

Thereupon,

$$F_Q(Q) = F_P(P) \quad (11)$$

And/or:

$$F_Q(Q) = \text{Prob}[P \leq f^{-1}(Q)] \rightarrow F_Q(Q) = F_P[f^{-1}(Q)] \quad (12)$$

If CN is specified accurately, the equation (4) and (5) will be found real for each pair of P and Q. In other words, observed rainfall frequency distribution and reverse function probable levels of the observed runoff will be equaled. Optimal CN will be calculated with trial and error. In this relationship, maximum error is specified between rainfall distribution function and minimum reverse distribution function and curve number is calculated appropriately with the aim of minimizing the obtained error. In fact, curve number is considered as a parameter of runoff reverse distribution function.

Monte Carlo Simulation

A probabilistic model is used for producing random variables within 0 and 1 range in Monte Carlo Simulation. The specified data will be compared with the data probable distribution function and its numerical amount is estimated. In this study, Monte Carlo Simulation has been used to study effects of probable distribution function, selective samples size and number of experimental curves. Selective initial experimental CNs (CN_u) and random samples size (N_n) are Monte Carlo Simulation parameters. According to the previous recommendations (SCS 1972), samples of $N_n = 5, 10, 25, 50, 75, 100, 200$ and 500 are used for producing random numbers. According to the proposal of the U.S. Soil

Conservation Service, initial experimental CN changes domain (45, 55, 65, 75, 85 and 95) has been used.

In the beginning, large rainfall matching (according to rainfall and runoff statistics) were selected and arranged in descending order and then, their cumulative distribution function ($F_p(P)$) is specified according to Weibull Method. After that, random numbers, which had been distributed uniformly, are produced within the range of 0 and 1. Each random number represents possible level of a rainfall event which is compared with the probability levels of rainfall and runoff heights as observed in Weibull Model. The rainfalls produced with each CN_u in Equations 2 and 3, are used to calculate amounts of runoff, then the cumulative distribution functions of runoff groups are specified.

The rainfall and runoff are produced in selective sample size through the use of rainfall distribution function and runoff distribution functions for each CN_u .

The criteria, as presented by Hawkins and et al. (1992) are used to study hydrological capability of the produced pairs of rainfall and runoff as follows:

- A) If runoff height, obtained from the produced rainfall, exceeded, this pair is removed.
- B) The "S" values are calculated according to the following relation which is the same as equation 2: $S = 5(P+2Q - \sqrt{4Q^2 + 5PQ})$. The values are calculated that calculated "S" value is placed within the criterion as presented by Hawkins and et al. ($P/S > 0.465$). Otherwise, the calculated pair is removed and new random numbers are produced, then frequency distributions are calculated. Depending on the number, called as " N_n ", rainfall and runoff data pair is necessitated with the aim of studying effects of the number of samples.

With the aim of estimating optimal CN, it seems necessary that initial curve number (CN_u) should be studied in more accurate intervals. Therefore, number of new experimental curves (CN_{trial}) is used in 0.1 intervals ($0.1 + CN_u = CN_{trial}$). For each sample size, N_n is determined with placing Q_u and CN_{trial} is set in equations 4 and 5 of P_r and $f^{-1}(Q)$ values.

With arranging values " P_r " in descending form, their cumulative possible levels and/or reverse potential distribution function is estimated as follows: $F_p(f^{-1}(Q))$.

Here, this method is carried out opposite SCS method. That is to say that with considering a curve number, which none study has been made for its optimal selection, cumulative potential distribution of rainfall is specified.

Kolmogrov-Smirnov Test has been used for studying reverse potential distribution functions. As a matter of fact, maximum existing error (D_{k-s}), between possible cumulative levels of real rainfall of the basin ($F_p(P)$) and cumulative possible levels of reverse functions $F_p(f^{-1}(Q))$, produced rainfalls (P_r) are calculated.

Thereupon, the test will be considered presentation of a curve number for N_n and CN_u which is called "Derived distribution Curve Number (CN_{dd})". Optimal CN is set from among the values.

Two other methods are used for comparison. Average Curve Number (CN_{ave}) is obtained from SCS runoff equation through the use of "S" average (as independent variable).

Median Curve Number (CN_{MED}) is calculated from SCS runoff equation of "S" median amount as independent variable. Of course, calculations are used for rainfall and runoff pairs with the aim of comparison.

RESULTS

Generally, 61 rainfalls and runoff data were selected out of total statistics extant in Kasilian and also 43 rainfall data were selected out of total statistics extant in Emameh watershed. Hjelmfelt and Hawkins (1984) criterion is used for the initial test in selecting larger events. With due observance to the ratio Q/P and P/S, we use the data which their runoff production is 100%. In fact, these data are the data which enjoy enough time continuity for producing runoff and represents physical condition of the basin. Figure 1 shows the way of determining optimal CN in Kasilian Basin with approaching cumulative frequencies of occurrence to each other. Each of these dash lines shows an experimental CN_u . The right-hand dash line is related to the assumed CN (80) while left-hand dash line is related to the CN (90). This diagram is related to the data measured in the basin. The line, which is approached from the right hand to the rainfall distributions, shows CN line (85) and has the least distance with the distribution of precipitation observed in Kasilian Basin. Namely, results of the least difference of cumulative experimental distributions have been observed with the different curves in cumulative distribution of CN (85). In Emameh Basin, the least difference of distribution of frequency is related to the CN (77). Therefore, with approaching frequency distributions of calculated rainfall modeling with the observed frequency distribution of rainfalls, it has been tried to select the most appropriate CN. Such activity is followed with lessening error in setting CN. In this error reduction, selection of larger events, measured in the basin, plays a very important role as well.

As it is observed in Figure 2 and 3, the more number of samples is increased; coefficient of changes will be decreased. For example, for a 500 samples, the coefficient of changes is neared zero. For 5, 10 and 25 sample sizes, changes will not follow specific rule especially with regard to CN_{dd} below than 45, 55 and 65.

This subject has been shown in different tests and in different executions of program. Variation coefficient of curves number was obtained for each sample size in repetitions of performing program. It should be noted that the "P" and "Q" pairs are used which enjoy one rainfall event in obtaining CN. Variability of CN_{MED} is high in number of lower samples but the way of CN_{MED} variability is identical at two basins, the reason of which is related to the removing effect of larger and smaller modeling in setting CN_{MED} .

Therefore, this method can be considered as the best alternative for stochastic method. Possibility of comparing initial assumed CN and the CN which offers stochastic method eventually, is one of the most important subjects in Monte Carlo Simulation. Variability of each pair of CN_u and N_u with CN_{dd} for 5 to 500-pair samples, representing Kasilian Basin, is shown in Figure 4.

Variation of curve number are high as compared with one-by-one line for the samples less of 5, 10, 25 and 50 pairs, but changes of curve number are not tangible for

the samples more than 100, 200 and 500 pairs. Thereupon, the more number of statistical samples is increased, the CN variability is decreased. Most points have not much more distance with one-by-one line in Emameh. High variability is related to $CN_u < 65$.

Figure 5 shows changes of initial curve number with the median curve number (CN_{MED}) according to stochastic method for basin representing Kasilian and Emameh basin respectively.

CM_{MED} has changes similar to CN_{dd} in Kasilian basin but these changes are more for smaller N_n while dispersion is high as compared with the individual line. On the other hand, it can be said that when the number of samples is found less, CN is overestimated. The mentioned issue is compatible with the Emameh Basin as well with this difference that overestimation of CN for CN_u which is smaller than 85 observed.

Box & Whisker Diagram is used to study CN_{dd} - CN_u changes, called as " D_{ud} (difference between underlined CN and derived distributed CN)" with the aim of studying changes of the duo better. For instance, Figures (7) and (8) and Box & Whisker Diagram, related to Kasilian and Emameh basin, has been presented. As it is observed in diagrams, the difference is reaches up to 26CN for $CN_u=45$. Although, this subject is found less for statistical courses i.e. for smaller N_n . Variation of curve number have been estimated fairly for $CN_u=95$ in such a way that changes is more observed in 8CN. The more number of statistical samples is increased, the curve number, which is obtained according to stochastic method, is more reliable. Although this doubt may be created that basin enjoys less curve number than the area above 85, stochastic method also enjoys capability of producing statistical data respect to the runoff frequency distribution. As it is observed in the diagrams, the more number of statistical courses is found less; the curve number of stochastic method has been overestimated especially for the small CNs. This subject is related to the insufficiency of samples for determining basic optimal CN.

The way of determining optimal curve number has been shown in Figure 9. D_{K-S} basin is obtained with any CN_{trial} or the curve number which is assumed as test and is placed in reverse function equation. A sample of this procedure has been shown in the Figure 9. K-S Test is used for studying and determining the least difference between cumulative frequency distributions. The obtained least difference was studied with CN_u and N_n matches for Kasilian and Emameh basins respectively.

- 1) If N_n is increased, D_{K-S} will be decreased in all CN_u . It indicates that the more number of samples is increased, more appropriate and logical results will be obtained and these results are obtained sooner.
- 2) Less CN_u has more D_{K-S} and curve numbers of 75, 85 and 95 have relative equal D_{K-S} . D_{K-S} will be decreased with the increased number of initial curves and number of samples. Using California, Hezen William, Gorton, Bloom and Chegodiou distribution did not show any remarkable difference in setting CN_{dd} . Although changes are observed in D_{K-S} rate, it does not leave any effect in the result.

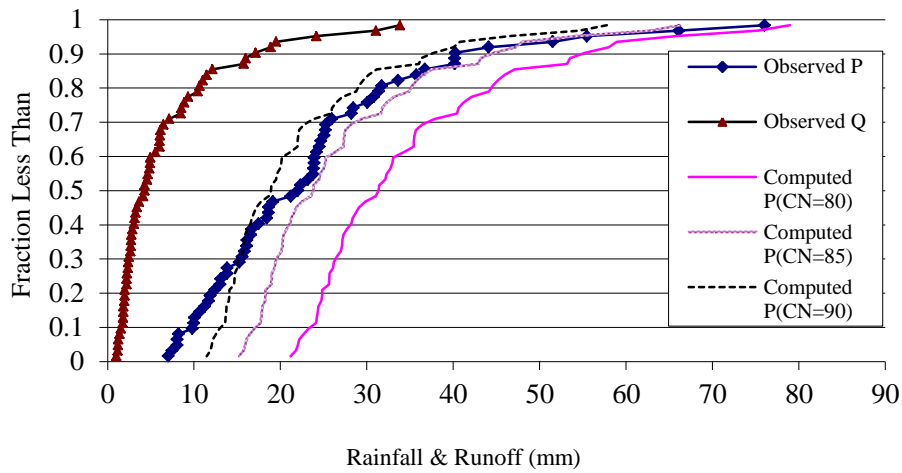


Figure 1. Way of Determining Optimal Curve Number in Stochastic Method for Kasilian Basin

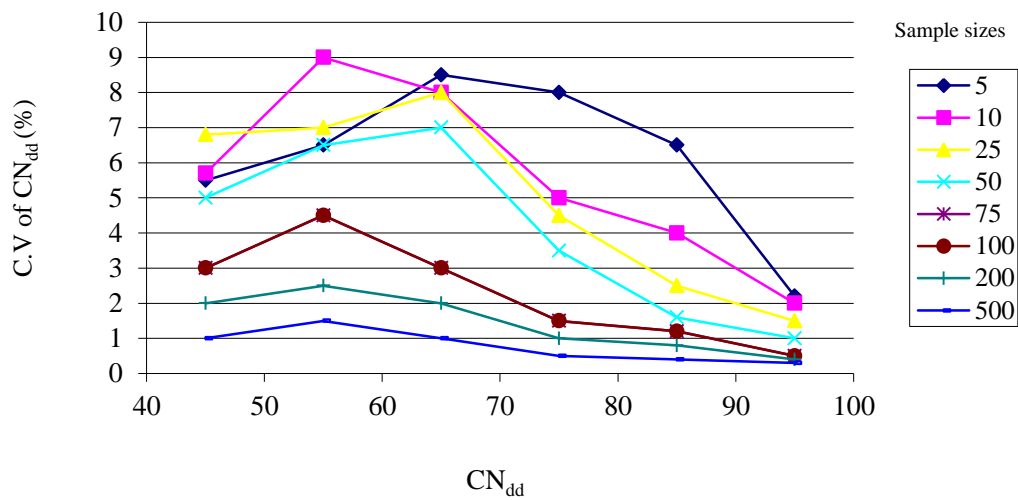


Figure 2. Comparing Coefficient of Variation for Derived Distribution Curve Number (CN_{dd}) in Stochastic Method (Kasilian Basin)

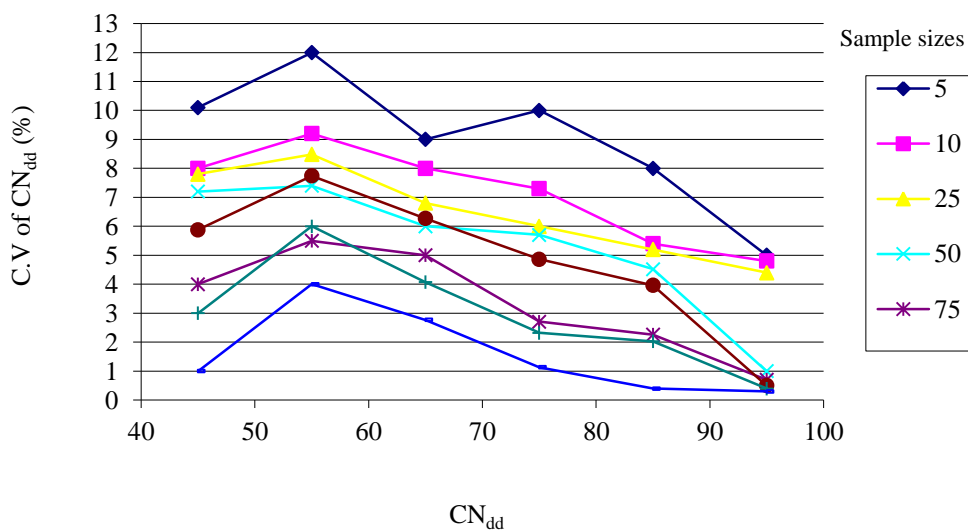


Figure 3. Comparing Coefficient of Variations for Determining CN_{dd} in Stochastic Method (Emameh Basin) Consequently, CN_{MED} of Emameh Basin is increased while facing basin stochastic CN

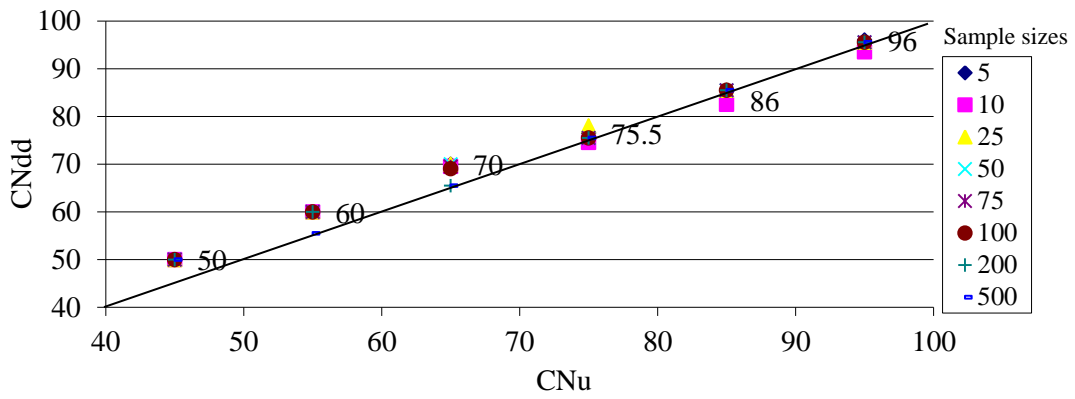


Figure 4. Relationship between Curve Number of Stochastic Method (CN_{dd}) and Initial Curve Number of Kasilian Basin for Different Number of Samples

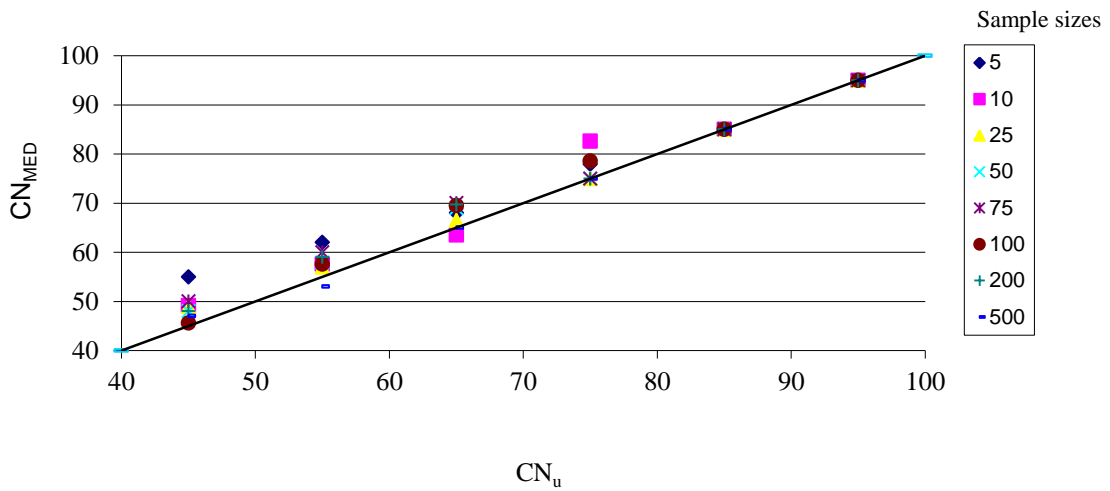


Figure 6. Relationship between Median Curve Number (CN Calculated Amounts) and Initial Curve Number of Kasilian Basin for Different Number of Samples

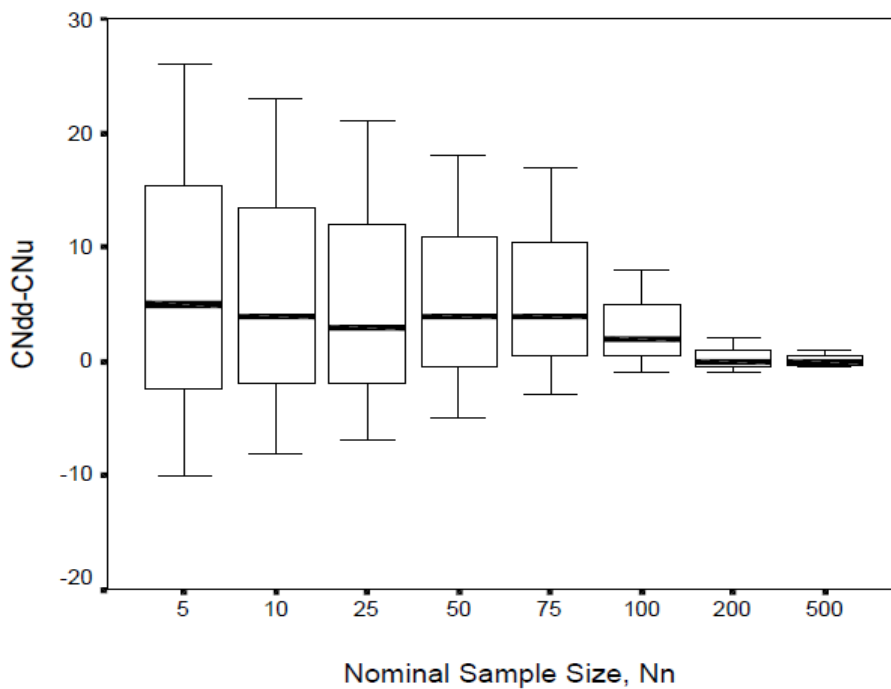


Figure 7. Box & Whisker Diagram, Experimental CN Difference with Optimal CN in Kasilian Basin ($CN_u=45$)

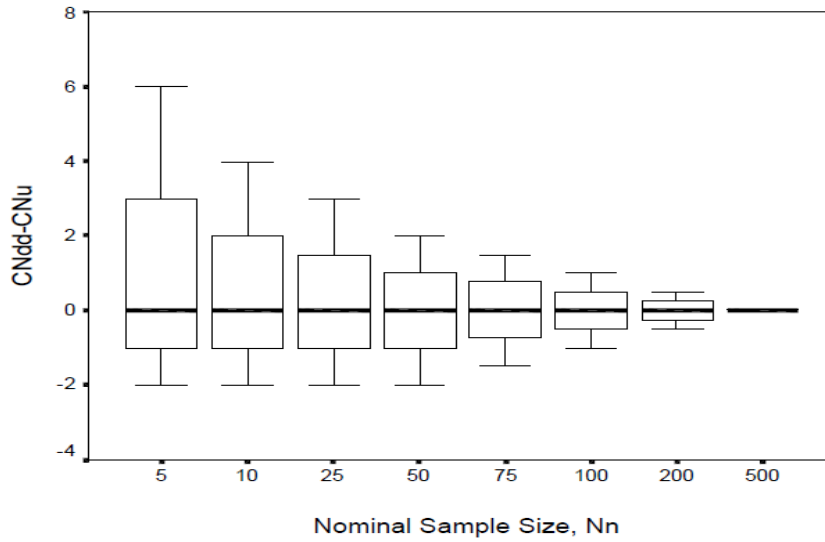


Figure 8. Box & Whisker Diagram, Experimental CN Difference with Optimal CN, in Emameh Basin ($CN_u=95$)

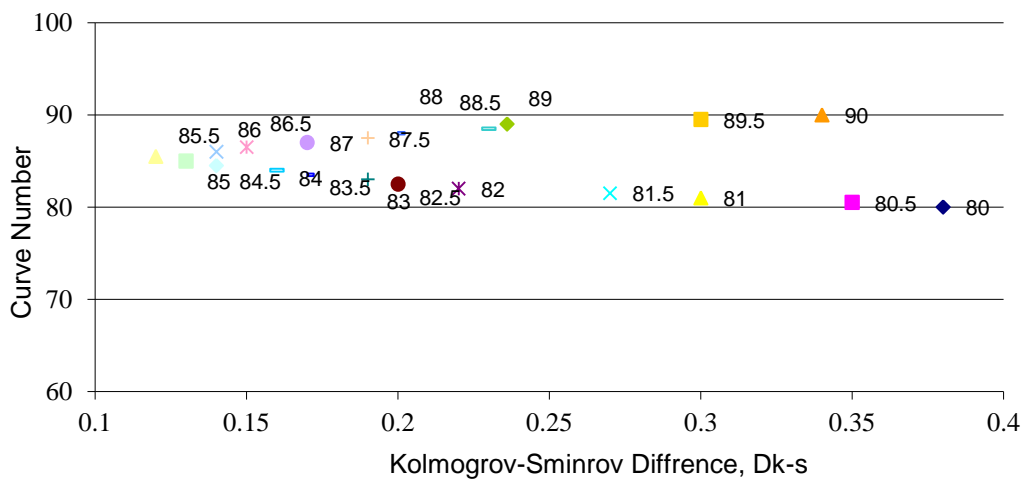


Figure 9: Way of Determining the Least Difference in Kolmogrov- Smirnov Test for $CN_u=85$, $N_n=10$

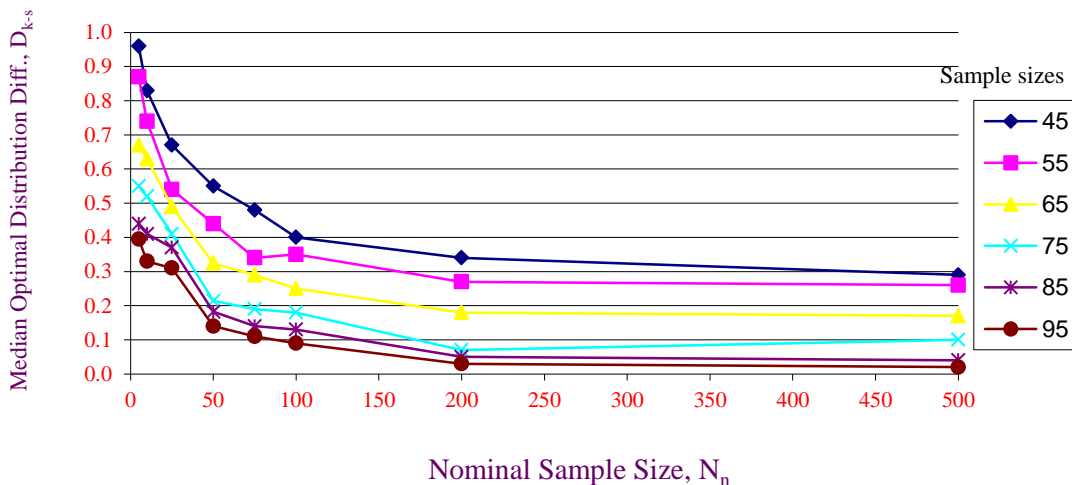


Figure 10. The Least Difference Variation between Cumulative Frequency Distribution (D_{K-S}) with Number of Samples for Different CN_u (Emameh Basin)

Table 1. Comparing Calculated Curves Number

CN_o	$CN_{asymptotic}$	CN_{AVG}	CN_{MED}	D_{K-S}	CN_{dd}	Modeling Samples	Basin
81	74	82.7	79	21%	78	61	Kasilian
77	68	83.9	72	29%	73	43	Emameh

According to Hawkins Method (1993), this method is used for determining CN through the use of rainfall and runoff values measured at the basin. Rainfall and runoff statistics are first selected through the use of Hawkins Criterion. After these matching, the selected modeling is arranged in order with the aim of having consistency of frequency matching. CN of each pair of “P” and “Q” is set. The relationship between CN and P is studied in coordinate axes. The result of curve fit to the obtained points show that tangent on this curve is $CN_{asymptot}$ when “P” moves towards infinite. Type of curve shows that it is placed in standard behavior group according to the classification of Hawkins and constant CN can be set in this basin which is equal to 81. In Emameh Basin, the curve number cannot be set according to the above method due to lack of constancy of the curve number.

DISCUSSION

The CN values, which are set in this study through the data measured in the basin, have been mentioned summarily in Table 1. It should be noted that CN_{dd} amount is found more in Kasilian Basin as compared with the other CNs. It may seem that stochastic method overestimates CN value in Kasilian Basin. But regarding Emameh Basin, this subject is completely opposite. The difference between CN_{dd} and CN_{MED} and/or CN_{AVE} and/or $CN_{asymptot}$ (land percent method) is meaningless in Kasilian Basin but these differences are meaningful and significant among CNs of Kasilian Basin.

In CN stochastic method, type of runoff rainfall distribution function is not found determinant. Although it is proposed to study this subject for other basins in estimating stochastic, using rainfall and runoff scattering data of a specific basin, which is not related to each other, is found possible in stochastic method. Mentioned issue is one of advantages of this method. No need to a lengthy-time statistics is the other point that should be taken into consideration. Also, using statistics of matching, requiring 24-hour time or more, is permitted.

Generally, stochastic method can be used for the other basins in Iran which have short-term statistics provided that distribution of frequency of runoff can be estimated in that area, in which, this issue is possible according to the hydrological methods. It is proposed to use this method for other basins for determining Curve Number (CN) with the aim of studying difference of its performance. With due observance to the significance of determining accurate CN in hydraulic and hydrological projects, it is necessary to set CN for all watershed basins. Moreover, restricting factors, such as lack of statistics and uncertainty to the extant meteorological and hydrological statistics, should be removed through use of statistical, mathematical and hydrological methods, details of which should be applied accurately.

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