

Investigating the Distribution and Orientation of Steel Fiber Reinforced Concrete

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ABSTRACT: The distribution functions of fiber location variables were investigated that can be used to produce an analytic model to predict the mechanical properties of SFRC. Considering the effect of surrounding boundaries and distribution function, theoretical expressions were derived for the number of fibers per unit cross sectional area in SFRC. A comparison between empirical results and the theory were done to confirm the analytical results. Fibers are usually used in concrete to control cracking due to both plastic shrinkage and drying shrinkage. The amount of fibers added to a concrete mix is expressed as a percentage of the total volume of the composite (concrete and fibers), termed volume fraction (Vf). Vf typically ranges from 0.1 to 3%. Fibers with a non-circular cross section use an equivalent diameter for the calculation of aspect ratio.

Key words: steel fiber reinforced concrete; steel fibers; vibration, distribution function.

ORIGINAL ARTICLE

INTRODUCTION

One of the important functions of steel fibers in concrete is to increase the tensile strength by decreasing the micro cracks developing under external load effects. The investigations show an evaluation of the correlation between tensile strength and fiber spacing or the average number of fibers per unit cross-sectional area. The tensile strength ratio of fibrous to plain matrix is slightly better correlated with the number of fibers per unit cross sectional area in comparison with the fiber spacing.

The number of fibers per unit cross-sectional area in concrete can be calculated through the following equation (Soroushian and Lee 1990):

$$N_f = \alpha V_f / A_f \quad (1)$$

Where N_f = number of fibers per unit area, V_f = volume fraction of steel fiber in concrete, A_f = cross sectional area of steel fiber and α = orientation factor.

The number of fibers per unit area at a section through the fibrous material is introduced as an alternative measurement representing the effectiveness of fibers in concrete. The orientation factor represents the average ratio, for all possible fiber orientation of the projected fiber length in the tensile stress direction to the fiber length itself.

Background

According to a number of studies, the orientation factors in the post-cracking behavior have been obtained within the range of $1/3$ to $2/\pi$ for a random 2-dimensional fibers arrangement and $1/6$ to $1/2$ for a random 3-dimensional arrangement using different assumptions (Zandi, 2006; Debicki et al., 1994). Krenechel 1964 has derived the fiber orientation factor as $3/8$ for a random two-dimensional arrangement of

fibers. The orientation factor may change as the matrix cracks and the fibers bridging the crack tend to align themselves to carry loads. Aveston, et al. 1974 have used a value of $1/2$. Laws, et al. 1974 take into account the contribution caused by the length effects and the effect of bond strength on the increase in the fiber alignment orientation factor.

In the coordinate system as shown in (Figure.1), α_z can be derived by the average projection in the first quadrant along Z direction. The equation given in the literature to derive the orientation factor of fibers is of the form (Soroushian and Lee, 1990):

$$\alpha_z = \frac{\int_0^{\pi/2} \int_0^{\pi/2} \frac{L_f}{2} \cos\theta \cos\phi \, d\phi \, d\theta}{\int_0^{\pi/2} \int_0^{\pi/2} \frac{L_f}{2} \, d\phi \, d\theta} = 0.405 \quad (2)$$

Where L_f is length of the fiber

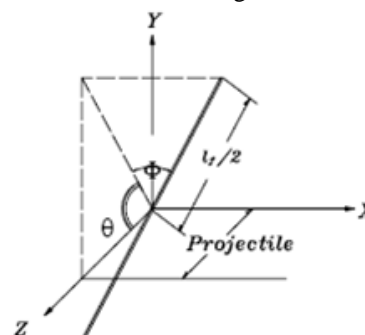


Figure 1. The coordinate system

Analytical investigation

The equation (2) is derived for a representative quadrant in space. It is assumed that ϕ and θ has a monotonous distribution. Obviously, in 3-D random

orientation a fiber has an equal probability of being oriented in any direction. However, calculation of distribution factor along X-axis (α_x) shows a greater value than the other distribution factors along other axes (α_y, α_z).

$$\alpha_x = \frac{\int_0^{\pi/2} \int_0^{\pi/2} \frac{L_f}{2} \sin\phi \, d\phi \, d\theta}{\int_0^{\pi/2} \int_0^{\pi/2} \frac{L_f}{2} \, d\phi \, d\theta} = 2/\pi = 0.64 \quad (3)$$

$$\alpha_y = \frac{\int_0^{\pi/2} \int_0^{\pi/2} \frac{L_f}{2} \sin\theta \cos\phi \, d\phi \, d\theta}{\int_0^{\pi/2} \int_0^{\pi/2} \frac{L_f}{2} \, d\phi \, d\theta} = 0.405 \quad (4)$$

Thus the assumption of a monotonous distribution for ϕ and θ is not corrects.

In this investigation, it is found that in order to equalize orientation factor in any direction a density function should be used or a two Uniformly Distributed Variable should be found.

Density function

Considering the equation $\alpha_y = \alpha_z$ and the independence of θ and α_x , it is assumed that θ has a monotonous distribution and then the density function of $\phi(F(\phi))$ is found as below:

$$\alpha_x = \alpha_z \Rightarrow \frac{\int_0^{\pi/2} \int_0^{\pi/2} \sin\phi F(\phi) \, d\phi \, d\theta}{\int_0^{\pi/2} \int_0^{\pi/2} F(\phi) \, d\phi \, d\theta} = \frac{\int_0^{\pi/2} \cos\phi \cos\theta F(\phi) \, d\theta \, d\phi}{\int_0^{\pi/2} d\theta \int_0^{\pi/2} F(\phi) \, d\phi}$$

Thus the density function will be calculated by the following equation:

$$\Rightarrow F(\phi) = \cos\phi \quad (5)$$

Leading to: $\alpha_x = \alpha_y = \alpha_z$

Uniformly distributed variables

In order to have a monotonous distribution without using a density function, it is also possible to use a special coordinate system (ρ, θ, x). In this coordinate system, the orientation factors are calculated as follows:

$$\alpha_x = \frac{\int_0^{L_f/2} x \, dx}{\int_0^{L_f/2} dx}$$

$$\alpha_z = \frac{\int_0^{L_f/2} \int_0^{\pi/2} \sqrt{\left(\frac{L_f}{2}\right)^2 - x^2} \cos\theta \, d\theta \, dx}{\int_0^{L_f/2} \int_0^{\pi/2} \frac{L_f}{2} \, d\theta \, dx}$$

$$\alpha_y = \frac{\int_0^{L_f/2} \int_0^{\pi/2} \sqrt{\left(\frac{L_f}{2}\right)^2 - x^2} \sin\theta \, d\theta \, dx}{\int_0^{L_f/2} \int_0^{\pi/2} \frac{L_f}{2} \, d\theta \, dx}$$

Assuming x and θ have a monotonous distribution, the same value for orientation factor in any direction is obtained that confirms the assumption. x evaluated with $\sin\phi$ in pherical coordinates so $\sin\phi$ has a monotonous distribution and $\cos\phi$ is a density function for ϕ .

Effect of boundary condition

When uniformly dispersed in an infinitely large volume of concrete, steel fibers are expected to have the equal orientation factors in any directions and the effect of boundaries is not considerable. In the presence of relatively close distance boundaries with respect to the fiber length, the orientation factor will become greater. According to the condition (2D, 3D), the orientation factor can be obtained as follows:

Fiber orientation factor in 2-D condition

In a pure 2-D distribution (Figure.2), with two boundaries', the following equation can be used in order to to derive the orientation factor:

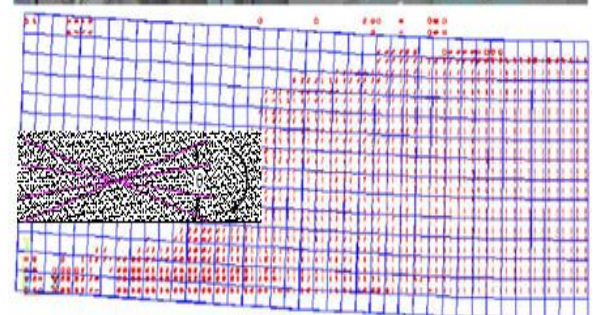
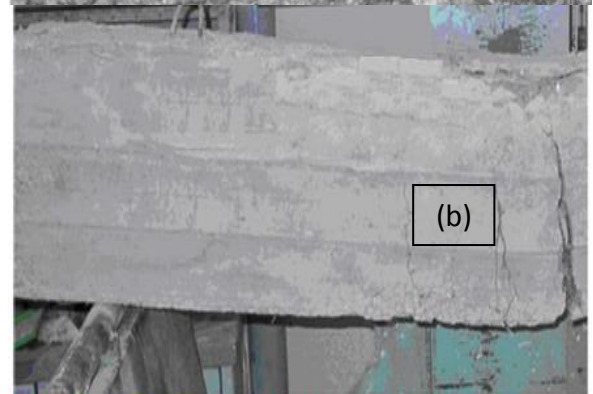
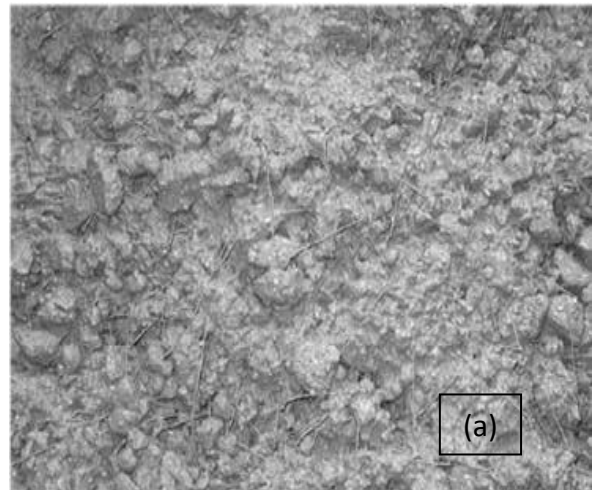
Fiber orientation factor in 3-D condition

Effects of two and four boundaries condition on the orientation factor of fibers are considered as follows:

A-Two boundaries

$$\alpha_{2D} = \begin{cases} \frac{\int_{-b/2}^{b/2} \beta_{2D} \, dx}{\frac{b}{2}} & \text{for } b \leq L_f \\ \text{for } b > L_f & + \left(1 - \frac{L_f}{b}\right) 0.64 \frac{L_f \int_{-b/2}^{L_f/2} \beta_{2D} \, dx}{\frac{b}{2}} \end{cases}$$

$$\beta_{2D} = \frac{\int_0^{\gamma} L_f \cos\theta \, d\theta}{L_f \int_0^{\gamma} d\theta} \quad \gamma = \sin^{-1}\left(2 \frac{x}{L_f}\right)$$



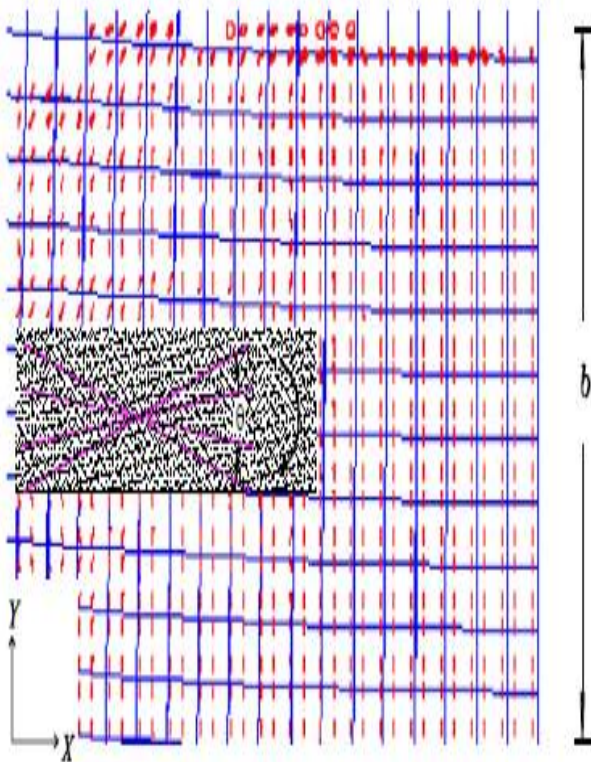
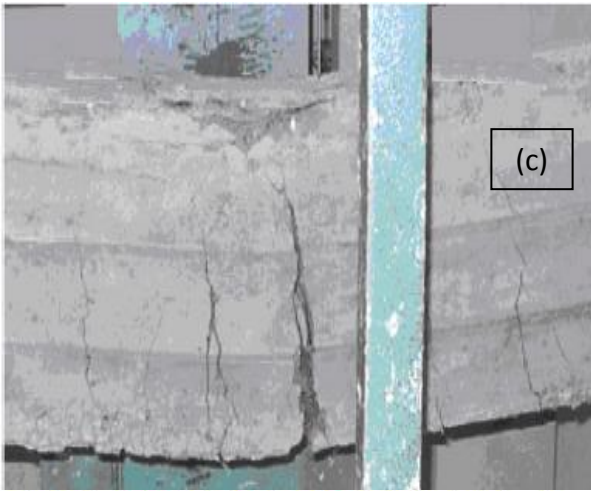


Figure 2. Two dimensional boundaries

Theoretical values versus experimental measurements

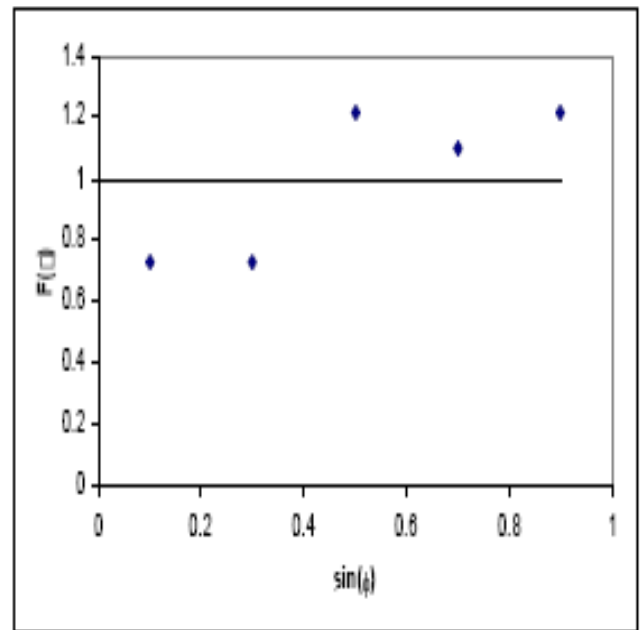
Two individual comparisons were done between the empirical results and the theoretical expressions.

A) Experimental works: Some cubic (15cm) and cylindrical (15/30) specimens were tested for this study. The concrete specimens had a water/cement ratio 0.22, cement content of 750 kg/m^3 , silica fume/cement ratio of 0.20, and super plasticizer content of 0.1% by cement weight. The fiber volume fractions were 1.0, 1.5, and 2.0 percent. The specimens were vibrated externally and tested after 28 days of air curing. The specimens have been separated into two parts by using a Brazilian testing method.

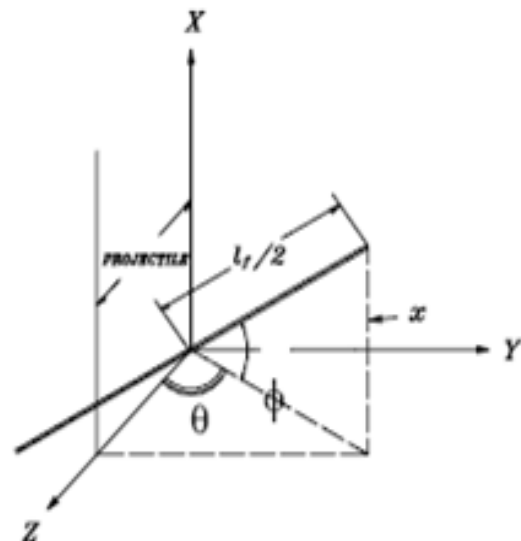
Two photographs were taken from each specimen with a non-metric camera (Figure.4). The photos were scanned with a metric scanner "ULTRA 5500", the digital images were processed with digital close photogrammetric software. Finally, 3D information of

each fiber in a 3D Cartesian coordination system was derived.

According to the results, it is observed that the variation of θ does not have a monotonous distribution. However, the values of θ and \sin follow a monotonous distribution (Figure.3). It should be mentioned that the new coordination system is assumed to neglect the effect of external effects such as fibers weight and vibration. The difference between the theoretical and measured value of the orientation factor of steel fiber inside concrete is not considerable. The effect of boundary conditions should be omitted by considering a distance of $L_f/2$ from each edge of the specimens.



(a)



(b)

Figure 3. (a) The monotonous distribution is shown as the solid line and the test results as the data points (b) The assumed coordination system.

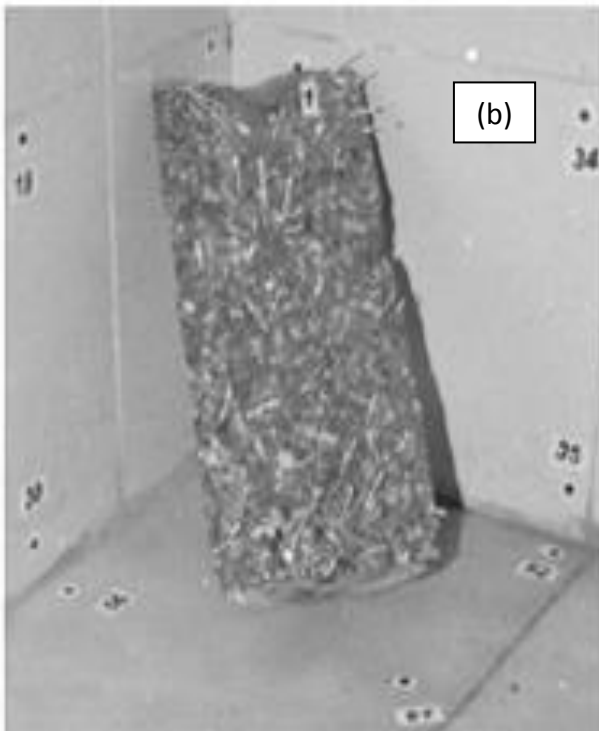
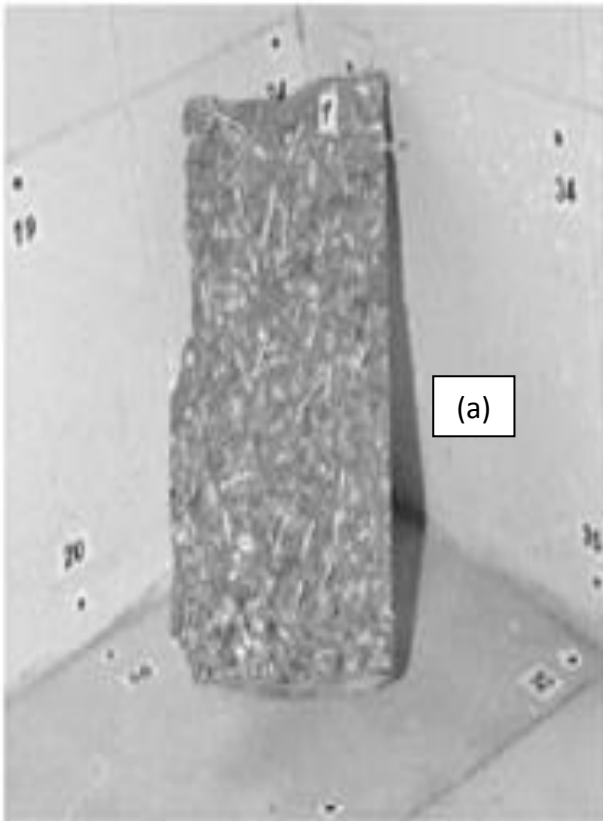


Figure 4. (a) cylindrical sample first view, (b) cylindrical sample second view

B- Comparison to database: The theoretical value of orientation factor (considering the four boundaries) is compared to the empirical results expressed in reference 2. In this study, measurements were categorized as top, middle and bottom (Figure.5), and orientation factor has been obtained from Eq.1. The results are summarized in table 1. The theoretical value

of orientation factor obtained from Eq.6 for the parameters chosen (fiber length of 2 in. and cross-sectional dimensions of 6 by 6 in.) 0.581 that is more accurate than the theoretical value obtained in the reference (0.537) (Soroushian and Lee 1990). The measured mean value of fiber (0.615) is closed to the calculated value of 0.581.

Table 1. Mean values of fiber orientation at different location on cross section and for different fiber types (Soroushian and Lee, 1990).

Location		Fiber type (No. of specimen)		
		Straight (16)	Hooked (3)	All (19)
Top	Standard Deviation	0.230	0.210	0.228
	Mean	0.610	0.606	0.610
Middle	Standard Deviation	0.238	0.772	0.220
	Mean	0.585	0.474	0.571
Bottom	Standard Deviation	0.242	0.216	0.238
	Mean	0.656	0.779	0.670
All	Standard Deviation	0.236	0.178	0.230
	Mean	0.615	0.616	0.615

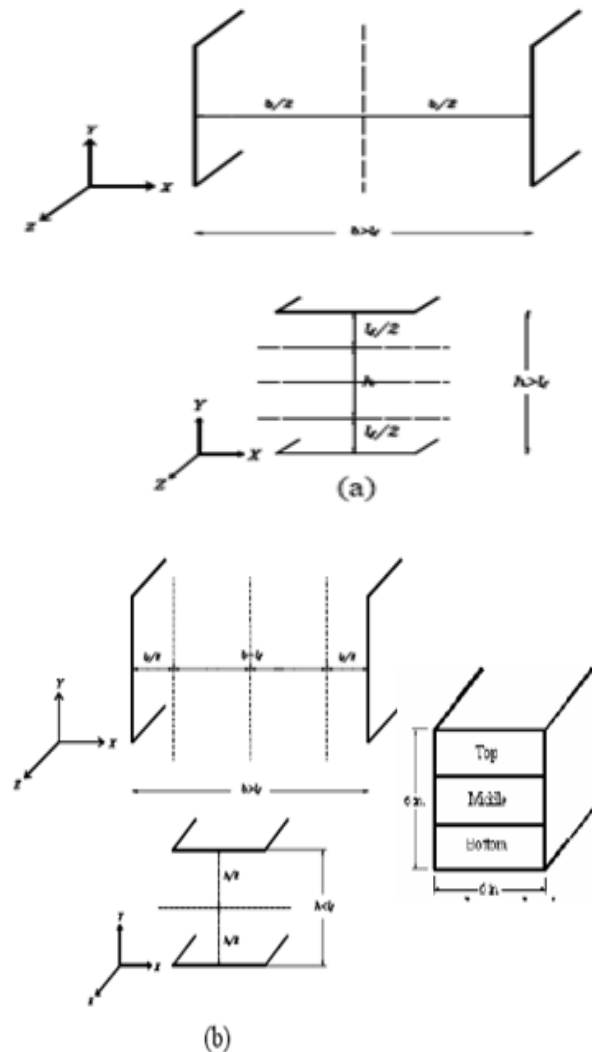


Figure 5. Measurement location

DISCUSSION AND CONCLUSION

The number of fibers per unit area (N_f) at a section through the fibrous material is introduced as an alternative measurement representing the effectiveness of fibers in concrete. The researches show an evaluation of the correlation between tensile strength and the average numbers of fibers per unit cross-sectional area. N_f is related to the fiber orientation factor. They also reduce the permeability of concrete and thus reduce bleeding of water. Some types of fibers produce greater impact, abrasion and shatter resistance in concrete. Generally fibers do not increase the flexural strength of concrete, and so cannot replace moment resisting or structural steel reinforcement. Indeed, some fibers actually reduce the strength of concrete.

The following conclusions were derived from the results of this investigation:

1-In order to equalize orientation factor in any direction, a density function ($F(\phi)$) is used or a two uniformly distributed variable is found. The empirical results confirm the theory.

2-Several investigations were precisely done, assuming that cross-sectional boundaries are the only factor disturbing the 3-D random orientation of fibers.

3-Considering four boundaries, theoretical value of orientation factor is compared to a database that is more accurate than the theoretical value obtained in the reference.

Acknowledgements

This work was supported by Karadeniz Teknik Üniversitesi laboratory of construction materials for this research is acknowledged.

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