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ORIGINAL ARTICLE

Sensitivity Analysis of Movable Bed Roughness Formula in Sandy Rivers

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ABSTRACT: Sensitivity analysis as a technique is applied to determine influential input factors on model output. Variance-based sensitivity analysis method has more application compared to other methods because of including linear and non-linear models. In this paper, van Rijn's movable bed roughness formula was selected to evaluation because of its reasonable results in sandy rivers. This equation contains four variables as: flow depth, sediment size, Bed form height and Bed form length. These variable's importance was determined using the first order of Fourier Amplitude Sensitivity Test. Sensitivity index was applied to evaluate importance of factors. The first order FAST based sensitivity indices test, explain 90% of the total variance that is indicating acceptance criteria of FAST application. More value of this index is indicating more important variable. Results show that bed form height, bed form length, sediment size and flow depth are more influential factors with sensitivity index: 32%, 24%, 19% and 15% respectively.
Key words: Sensitivity Analysis, Variance, Movable bed roughness formula, Sandy River.

INTRODUCTION

Once the critical shear stress on the bed in unidirectional flow is exceeded, the sediment particles forming the bed are transported at a rate, which increases with increase in shear stress on the bed. The bed in the process remains plane under some conditions, but under other conditions develops transversely oriented bed features known as ripples, sand waves or dunes, and anti-dunes as shown in Figure 1.

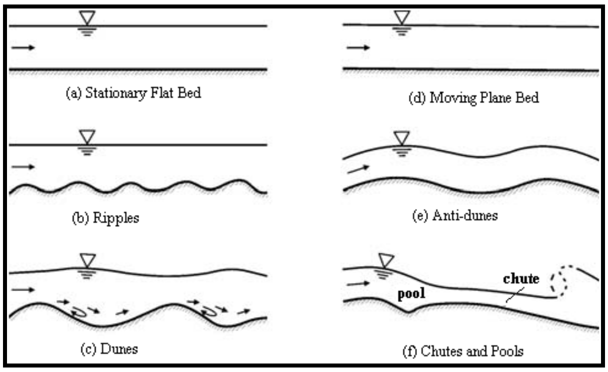

Figure 1. Bed forms in alluvial rivers

These bed-forms travel beneath the flow, take part in the sediment transport, and govern the relationship between flow velocity, flow depth and slope. In other words, they affect the friction and sediment transport. They also leave back a characteristic imprint in the enclosed deposits (Garde, 2006). For a channel bed with sand grains and bed forms (such as sand ripples and dunes), the bed resistance may be divided into the grain (skin or frictional) shear stress and the form shear stress. Therefore, using an equation with more accuracy to evaluate flow resistance will cause acceptable results in practical aspects. There are various formulas to estimate bed roughness in movable bed. Einstein and Barbarossa (1952), Engelund and Hanssen (1967), Alam and Kennedy (1969) proposed empirical methods for separately calculating the grain and form resistance to flow. Li and Liu (1963), Richardson and Simons (1967), and Wu and Wang (1999) suggested direct calculation of the total roughness coefficient of a movable bed. Van Rijn (1984c) and Karim (1995) established empirical relations to predict the height of bed forms and then the roughness coefficient on a movable bed. Brownlie (1983) proposed a formula to determine the flow depth rather than the roughness coefficient in an alluvial river. The movable bed roughness formulas of Li and Liu (1963), Van Rijn (1984), Karim (1995), Wu and Wang (1999) were tested against 4376 sets of flume and field data collected by Brownlie (1981). These data sets were measured by many investigators in several decades, covering flow discharges of 0.00263–28825.7 m3s−1, flow depths of 0.04–17.3 m, flow velocities of 0.2–3.32 ms−1, bed slopes of 0.00002–0.067, sediment median diameters of 0.011–76.1 mm, and sediment size standard deviations up to 9.8. Table 1 compares the measured and predicted flow depths.

Table 1. Comparison of measured and predicted depths

Error range	% of calculated flow depths in error range			
	Li-Liu	Van Rijn	Karim	Wu-Wang
±10%	21.8	44.0	41.0	41.5
±20%	41.8	77.9	74.9	75.9
±30%	58.8	91.4	91.0	94.4

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It can be seen that the van Rijn, Karim, and Wu-Wang formulas almost have the same level of reliability for predicting the flow depth (Wu, 2008). As compared with the Li-Liu formula, the Wu-Wang formula has much improvement. But Van Rijn Formula has good agreement rather than the others. The Van Rijn formula has different parameters which should be known to evaluate flow resistance. In this paper sensitivity of each parameter on formula output has been considered using FAST variance based method.

Sensitivity analysis is a technique to study relation between input and output of a model. On the word, it is a method to assess relative importance of each input factor on the output variable (Schwinger, 2004). According to the vast application of SRM model, it is necessary to evaluate which input factor has more influence on the output of model.onm the other hand, if importance level of each factor be determined, it will help to modelers to simulate flow discharge, accurately. This causes more confidence for water management planning. There are different methods to do sensitivity analysis. Fourier amplitude sensitivity Test (FAST) is a technique, which works irrespective of the degree of linearity or additivity of model. The use of this method is proposed by Cukier (1973), Cukier et al. (1978) and Schaibly and Shuler (1973). Also Dawson et al. (2008), Hall et al. (2005), and Pappenberfger et al. (2008) have been used FAST in the field hydrodynamics.

MATERIALS AND METHODS

Cukier et al. developed variance based sensitivity analysis in the early 1970s. Their method is conditional variance based on first-order effects and is called Fourier Amplitude Sensitivity Test (FAST) is applicable to nonlinear models. In FAST, the variance V(Y) of Y is decomposed using spectral analysis, so that

$$V=V_1+V_2+...+V_n+r \qquad (1)$$

Where V_i is that part of the variance of Y that can be attributed to X_i alone and r is a residual (Cukier et al., (1973)). Saltelli et al. (2008) developed an algebraic equation as following:

$$V(Y)=E_{X_i}[V(Y|X_i)]+V[(E_{X_{\sim i}}(Y|X_i)] \qquad (2)$$

Where $V[(E_{X_{\sim i}}(Y|X_i)]$ is referred to as the variance of conditional expectation of Y givenX_i, the subscript $X_{\sim i}$ denotes the vector of all factors other than X_i, and $E_{X_i}[V(Y|X_i)]$ is mean expectation value of conditional variance over all possible values of factor X_i. The FAST sensitivity index is simply as following:

$$S_i=\frac{V[(E_{X_{\sim i}}(Y|X_i)]}{V(Y)}=\frac{V(Y|X_i)}{V(Y)}=\frac{V_i}{V} \qquad (3)$$

Can be taken as a measure of the sensitivity of Y with respect to X_i. In classic FAST, only the main effect terms V_i are computed, and the success of a given analysis is empirically evaluated by the sum of these terms (Saltelli et al., 2008). If this is high, as rule of the thumb greater than 0.6, then the analysis is successful. The V_i describe the so called "additive" part of a model and additive models are defined as those for which $\sum S_i=1$.

Note that in this context, the condition of "additivity" is more general than linearity, to which Standard Regression Coefficients (SRCs) are restricted. SRCs is linear regression based sensitivity analysis method which is applicable to linear models. The measure of $V(Y)-V\big(E(Y|X_{\sim i})\big)=E(V(Y|X_{\sim i}))$ (4)

Is the remaining variance of Y that would be left, on average, if it can be determined the true values of $X_{\sim i}$. The average is calculated over all possible combinations of $X_{\sim i}$ since $X_{\sim i}$ are uncertain factors and their 'true values' are unknown. Dividing by $V(Y)$ we obtain the total effect index for Xi:

$$S_{Ti}=1-\frac{V[E(Y|X_{\sim i}=x^*_{\sim i})]}{V(Y)} \qquad (5)$$

Whatever the strength of the interactions in the model, Si indicates by how much one could reduce, on average, the output variance if X_i could be fixed; hence, it is a measure of main effect. Whatever the interactions in the model, $S_{i1,i2,...,in}$ indicates by how much the variance could be reduced, on average, if one could fix X_{i1}, X_{i2}, X_{i3}, …, X_{in}. By definition, S_{Ti} is greater than S_i or equal to S_i in the case that X_i is not involved in any interaction with other input factors. The difference $S_{Ti}-S_i$ is a measure of how much X_i is involved in interactions with any other input factor. $S_{Ti}=0$ implies that X_i is non-influential and can be fixed anywhere in its distribution without affecting the variance of the output. The sum of all S_i is equal to 1 for additive models and less than 1 for non-additive models. The difference $1-\sum S_i$ is an indicator of the presence of interactions in the model. The sum of all S_{Ti} is always greater than 1. It is equal to 1 if the model is perfectly additive (Saltelli et al., 2008).

In this paper van Rijn (1984c) equation was considered for sensitivity analysis. Van Rijn (1984c) established a relation for the sand-dune height, Δ, as shown in Figure 2 and expressed as

$$\frac{\Delta}{h}=0.11\left(\frac{d_{50}}{h}\right)^{0.3}(1-e^{-0.5T})(25-T) \qquad (6)$$

Where T is the non-dimensional excess bed shear stress or the transport stage number, defined as $T=\left(\frac{U'_*}{U_{*cr}}\right)^2-1$; U'_* is the effective bed shear velocity related to grain roughness, determined by $U'_*=\frac{U\sqrt{g}}{C'_h}$ with $C'_h=18\log\frac{4h}{d_{90}}$; U_{*cr} is the critical bed shear velocity for sediment incipient motion, given by the Shields diagram; and d_{50} and d_{90} are the characteristic diameters of bed material.

In van Rijn's method, the length of sand dunes is set as $\lambda_d=7.3h$, the grain roughness is $3d_{90}$, and the form roughness is $1.1\Delta(1-e^{-\frac{25\Delta}{\lambda_d}})$. Therefore, the effective bed roughness is calculated by means of

$$k_s=3d_{90}+1.1\Delta(1-e^{-\frac{25\Delta}{\lambda_d}}) \qquad (7)$$

And the Chezy coefficient is then computed by

$$C_h=18\log\left(\frac{12R}{k_s}\right)=18\log\frac{12R}{3d_{90}+1.1\Delta(1-e^{-\frac{25\Delta}{\lambda_d}})} \qquad (8)$$

Where R is hydraulic radius. In this paper, Eq. (8) is named the model for simplicity. In order to explore the

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sensitivity of response of the model to variation in the input factors, the input factors (of number 4) were assigned the distributions in Table 2.

Table 2 Distribution of input factors for the SRM model

Variables	Distribution type	Input factors
h	Normal	h~(1-7)
d_{90}	Log-normal	d_{90}~(0.03-0.08)
Δ	Log-normal	Δ~(0.01-4)
λ	Normal	λ~(0.5-8)

The mentioned range for variables in table 2 are selected according to reasonable values of each one. Each factor was tested of number 5,000. Figures 3 to 5 show the result of model input for a range of values of (h,d_{90}), (Δ,d_{90}), and (Δ,λ), respectively. Table 3 shows the result of FAST about the model input variables.

Table 3 The first order FAST sensitivity analysis results

Variables	S_i	S_{Ti}	S_{Ti}-S_i
h	0.15	0.25	0.1
d_{90}	0.19	0.31	0.12
Δ	0.32	0.73	0.41
λ	0.24	0.45	0.21
Total	0.90	-	

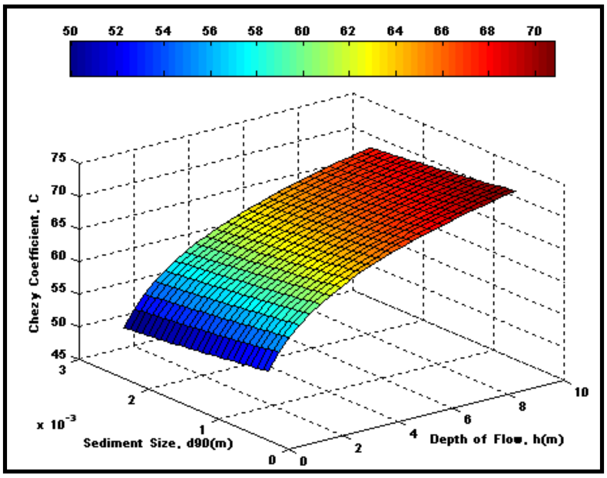

Figure 3. The model output as a function of h and d_{90}

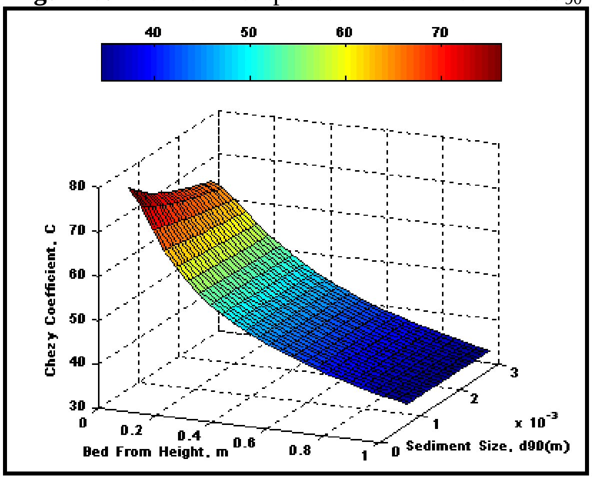

Figure 4. The model output as a function of Δ and d_{90}

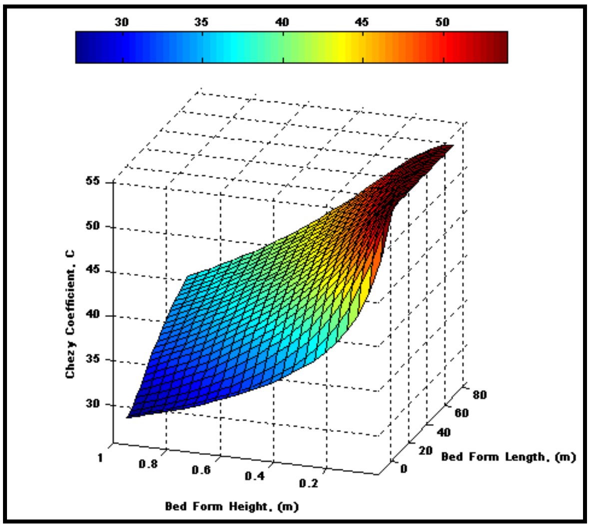

Figure 5. The model output as a function of Δ and λ

RESULT AND DISCUSSION

In this paper variance-based sensitivity analysis of movable bed roughness formula in sandy rivers was studied. There are various formula to evaluate bed resistance. Van Rijn's equation was selected according to the less error among existence equations. This equation contains four input variables which importance of each factor on equation output is the aim the paper. First, each variable was selected in its defined range. Five thousand numbers were considered for each input variables. Sensitivity index and remaining variance of each factor were calculated using equation (3) and (5), respectively. According to table 3, the first order FAST based sensitivity indices, explain 90% of the total variance. According to column 2 of table 3, ranking of influential variables are Δ, λ, d_{90} and h which are accounting 32%, 24%, 19% and 15%, respectively. These values are illustrated in Figure 6, which shows pie diagram of FAST indices.

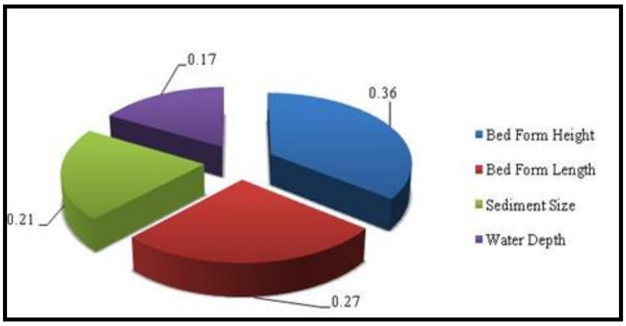

Figure 6. Variance decomposing of model output

Column 3 of table 3 illustrates that Δ and λ are two variables that are involved in interaction with any other input factor. Finally, it can be concluded that bed form properties, i.e. Δ and λ, are two main variables which have the most importance on model output. This result can be demonstrated from figures 3 to 5. As seen, the model output variation corresponding to flow depth is invisible than the figures 4 and 5. The effects of bed form properties on flow resistance are very tangible. On the other word, they need to be estimated more accurately.

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