

Assessment of the Topological Properties of Akure Road Network

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ABSTRACT: This paper assesses the topological properties of Akure road network using graph theory. Three 1,609-square meters of different urban street sections were sampled from Akure road network map and developed into graphs using the primal approach. The resulting graphs were then described by their adjacency matrices. Investigation into the topological properties of the resulting graphs revealed that the three networks produced possessed small average clustering coefficients of 0.007, 0.016, and 0.026 and small average path lengths of 0.066, 0.088 and 0.062. The paths lengths obtained deviated from one obtained when it was assumed to be randomized. The degree distribution obtained over the networks could not be approximated using either binomial or Poisson distribution. Further investigation revealed Akure road network exhibited a power law degree distribution.

Keywords: Network, graph theory, topology, clustering coefficient, path length, degree distribution, scale free network

ORIGINAL ARTICLE

INTRODUCTION

Networks are ubiquitous in nature and they describe various systems in whatever society they exist [1]. Nature, society and many technologies are sustained by numerous networks that are too important to fail but paradoxically for decades have also proved too complicated to understand [2]. Networks, especially transportation networks, are important for human life because they aid movement of people and goods from one place to another [3].

In addition, a safe and efficient land transportation system is an essential element of sustainable regional or national economy. Roads have been and continue to be the backbone of the land transportation network that provides accessibility for the required mobility to support economic growth and promote social activities [4].

Transportation networks are an integral part of infrastructure in many countries. Hence, the study of transportation networks is an important field [5]. These networks nowadays are a subject of intense research as modern society depends on them. However, complex web like structures describe a wide variety of systems of high technological and intellectual importance. These systems range from communication networks to transportation systems. In conventional transport analysis and modelling, a transport system is considered as a network comprising of a series of nodes and links [6]. The nodes are the elementary components of the system and the edges connect pairs of nodes that mutually interact thereby exchanging information [7].

Transportation networks inherently have a node and link structure where the links represent linear features providing for movement such as roads while the nodes represent intersections. The research and analysis which are conducted in the field of public transport require

building network models based on graph theory [8]. According to Graph theory deals with problems having graph structure [9]. Therefore, critical transportation infrastructure systems can be modelled as graphs/networks [10].

The main difference between many of the recently studied complex networks and transport networks lies in their spatial structure. Spatial networks are a special class of networks whose nodes are embedded in a two or three dimensional Euclidean space and whose edges do not represent or define relationships in abstract space but are real physical connections [11]. The interest in the spatial structure of transport networks has been driven by the inherent impact of the network structure on its performance and its effect on land use [12].

In the last decade, networks have been modelled as graphs in order to understand their topology which is concerned with the arrangement and connectivity of nodes and links within it [13]. A number of graphical network models have been developed namely random, regular, scale free and small world networks while some of the measures of network topology are the average path length, clustering coefficient and degree distribution [14]. However, different systems exhibit different topologies ranging from star topology to irregular patterns and an urban street network is often perceived as a graph whose edges represent street segments and vertices represent segment intersections [15]. It is in this regard that this study seeks to assess the topological properties of Akure road network using graph theory and to determine its structure based on its properties.

METHODOLOGY

The network analysis was based on the primal approach in which zero dimensional geographic entities

(intersections) are turned into zero dimensional graph entities (node) placed in two dimensional Euclidean space and one dimensional graph entities such as streets are turned into one dimensional graph entities [16]. Primal graphs are constructed by following a road-centreline-between-nodes rule where real intersections are turned into graph nodes and real streets are turned into graph edges.

Consequently, all the graph edges are defined by two nodes (the endpoints of the arc) and, possibly, several vertices (intermediate points of linear discontinuity). The intersections among edges are always located at nodes and the edges follow the footprint of real streets as they appear on the source map. The Akure road network map which was used for the study was obtained from the Ondo State Ministry of Works and Housing (OSMWH). Three sections were sampled from the map and developed into networks using graph theory. The sections represent three major locations/areas in Akure namely Oke-Aro (network 1), Sijuwade (network 2) and Oyemekun (network 3) (as shown in Figure 1).

The resulting edges were assigned with weights which represent the corresponding distance values between the intersections. Each graph, denoted by the symbol G , was described by the adjacency matrix $A = \{a_{ij}\}$, an $N \times N$ square matrix whose element a_{ij} is equal to 1 if (i, j) belongs to the set of links and zero otherwise. Some of the measures used to assess the topological properties of the graphs are as follows:

- **Path length**

This is defined as the typical separation between two generic nodes in a graph G which can be measured by the characteristic path length L defined as:

$$L(G) = \frac{1}{N(N-1)} \sum_{i,j \in G, i \neq j} d_{ij} \quad (2.1)$$

where d_{ij} is the length of the shortest path between nodes i and j .

- **Clustering coefficient**

This indicates the probability that two node neighbours are connected. For each node i of

G , we consider the subgraph G_i of its first neighbours that is obtained in two steps:

- Extracting i and its first neighbours from G ;
- Removing the node i and all the incident edges.

If node i has k_i neighbours, then G_i will have k_i nodes and at most $k_i \frac{(k_i-1)}{2}$ edges, C_i is proportional to the fraction of these edges that really exist and measures the local group cohesiveness of vertex i . C is the average of C_i calculated over all nodes:

$$C(G) = \frac{1}{N} \sum_{i \in G} C_i \quad (2.2)$$

and

$$C_i = \frac{2e_i}{k_i(k_i-1)} \quad (2.3)$$

where e_i is the number of edges in G_i .

- **Node degree**

This indicates the number of edges connected to or incident on each node in the network. The degree k_i of node i is defined as

$$k_i = \sum_{j \in G} a_{ij} \quad (2.4)$$

The average degree is

$$\langle k \rangle = \frac{1}{N} \sum_{j \in G} k_j = \frac{2K}{N} \quad (2.5)$$

The way the degree is distributed among the nodes is an important property of a network that can be investigated by calculating the degree distribution $P(k)$, i.e., the probability of finding nodes with k links. The degree distribution is defined as:

$$P(k) = \frac{N(k)}{N} \quad (2.6)$$

where $N(k)$ is the probability of finding a node with k links.

ANALYSIS AND INTERPRETATION OF RESULTS

Three 1-square mile sections of different urban street networks were sampled from a map of Akure with scale 1:20000. The sections were imported to GIS (Geographic Information System) environment and the primal graphs were constructed using node-centreline-approach-between-node formats (Figure 2). The areas studied within Akure exhibit striking differences in the activities that take place within and the level of land development. The basic structure observed over the planar road network (network embedded in real space) in Akure is branching network with no form of circuit network. Branching network is distinguished by the presence of tree-like structures which consist of connected lines without any complete circuit.

These graphs have the following properties:

- They are weighted
- They are sparse indicating that $K \leq \frac{N(N-1)}{2}$

where k and N are the numbers of edges (link) and nodes respectively

- They are connected meaning that there exists at least one path connecting any two vertices with a finite number of steps.

In this study, dead ends are considered as nodes since travel paths from such locations make use of the nearest or neighbouring nodes.

Analysis of Degree Distribution and Clustering Coefficient

Degree Distribution

Not all the nodes in a network under study have the same degrees. The spread in node degrees is characterised by the distribution function which gives the probability of a randomly selected node having a particular degree, k . The way the degree is distributed among the nodes is an important property of a network that was investigated by calculating and plotting the degree distribution $P(k)$ as shown in Figure 3.

An important observation in this analysis is that transportation networks are embedded in real space where

nodes and edges occupy precise positions in 2 dimensional Euclidean spaces. Edges in these networks –

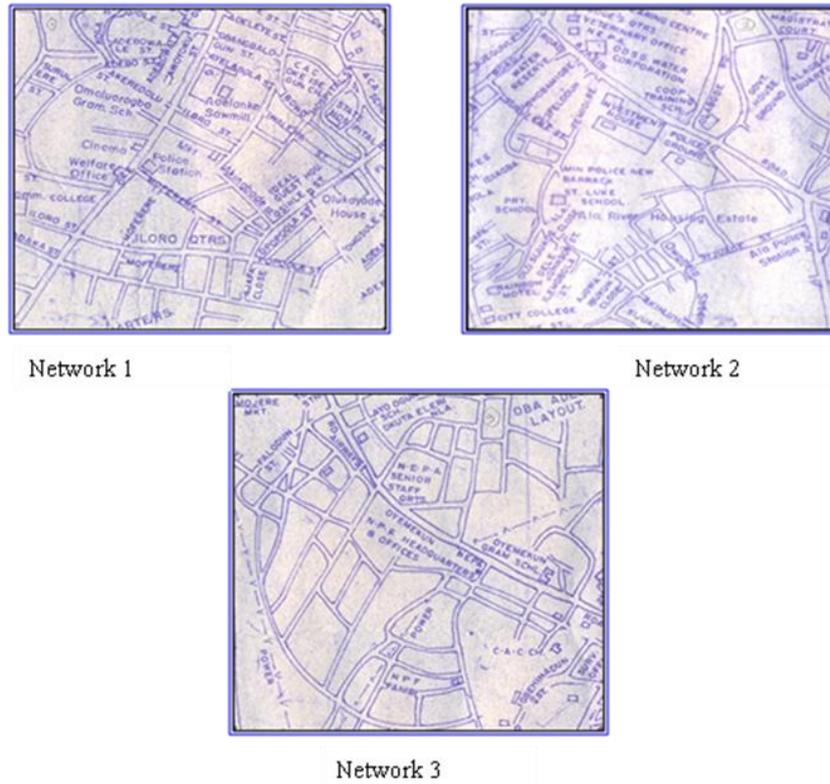


Figure 1. The three sections sampled from Akure street map (Scale - 1: 20,000 meters)

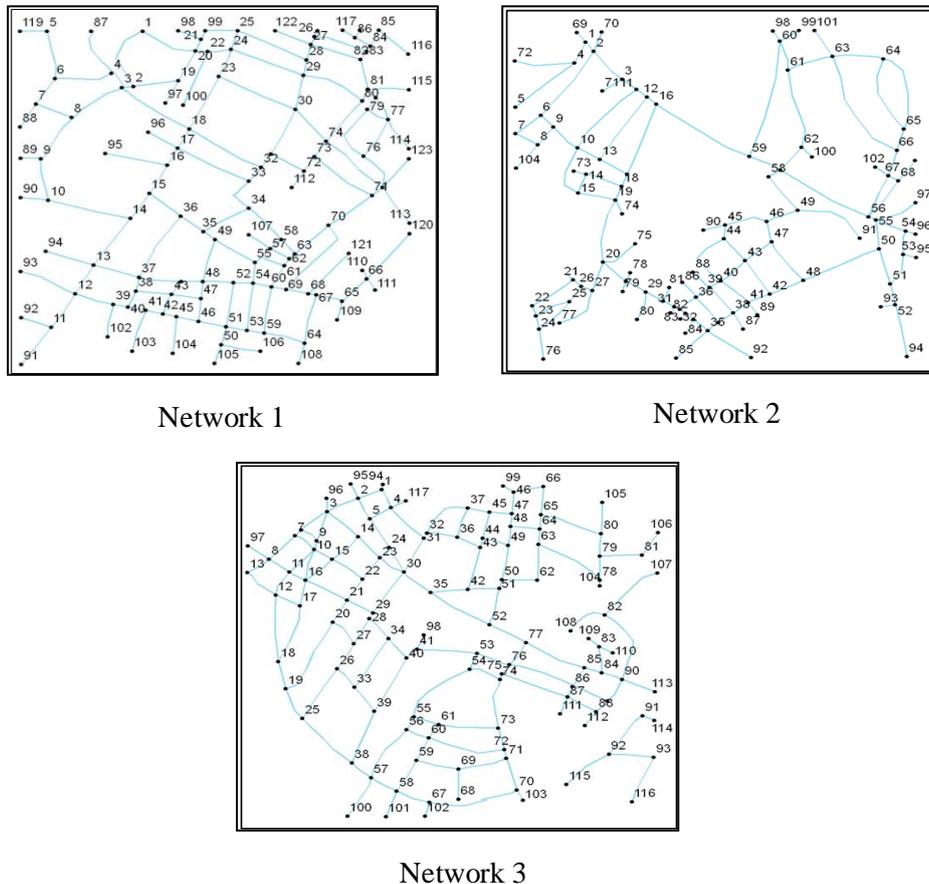


Figure 2. Primal Graph representations of the three sections

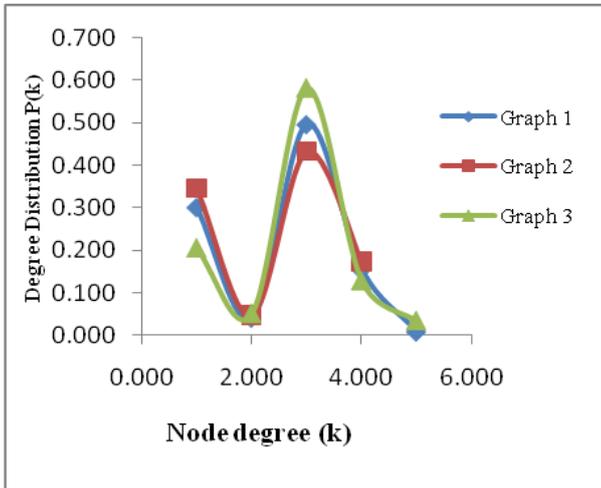


Figure 3. Probability distribution of node degrees over the three network case-studies.

– have physical constraints suggesting that spatial networks are strongly constrained and this has consequences on the degree and the number of long range connections. Analysis to classify the networks revealed that the networks do not exhibit the basic characteristic of random networks. Also, the probability distribution $P(k)$ cannot be approximated using either binomial distribution of the form in equation 3.1 or poison distribution of the form in equation 3.2.

$$P(k) = \binom{N-1}{k} P_{N-1}^k (1-P)^{N-1-k} \quad (3.1)$$

$$P(K) = \frac{e^{-\langle k \rangle} \langle k \rangle^k}{k!} \quad (3.2)$$

where $\langle k \rangle$ is the average degree, K is the node degree, $P(k)$ is the probability of finding a node with a particular degree and N is the number of nodes.

Investigation into the degree distribution of the road networks showed that they cannot be approximated by either binomial or poison distribution considering the average degrees obtained. This can be linked with the absence of links crossing without their point of crossing becoming a node. The degree distribution of a random graph is always peaked around the average degree $P\langle k \rangle$. In the case of the urban streets considered, average degrees (k) of 2.529, 2.433 and 2.718 were observed over networks 1, 2 and 3 respectively. The degree distributions obtained for the networks were peaked at 0.496 for network 1 (as shown in Table 1) and peaked at 0.433 and 0.581 for networks 2 and 3 respectively. This suggested that the networks are not random.

Due to the difficulties encountered in fitting the networks to either binomial or Poisson distributions, further investigation of the degree distribution $P(k)$ for the case of urban streets considered showed that it followed power law distribution (i.e. $P(k) = k^{-\gamma}$). The exponent of the power law obtained from graphs ranges between 0.638 to 4.621, 0.763 to 4.379, and 0.494 to 4.285 respectively for graph networks 1, 2 and 3. However the power law exhibited by these networks has no scale. This corroborates earlier research findings which showed that scale free networks grow with time and their links are attached to nodes preferentially [14].

Table 1. Node Degree (k), Probability Distribution of the Node Degree $P(k)$ and Exponent of the Power law (γ) for Network 1

Node degree (k)	Degree occurrence $N(k)$	Degree distribution $P(k)$	Poisson distribution	Power law (γ)
1	37.000	0.301	0.202	-
2	5.000	0.041	0.254	4.621
3	61.000	0.496	0.216	0.638
4	19.000	0.154	0.138	1.347
5	1.000	0.008	0.070	2.990

Clustering Coefficient

The clustering coefficient of a given node indicates the probability that the direct neighbours of a given node are directly connected. It signifies the interaction between the component nodes as well as the ability of the network to respond to link failure. Investigation into the clustering coefficient of the nodes revealed that most of the neighbours of a particular node selected are not highly connected.

When a node is at the centre of a fully interconnected cluster, the clustering coefficient is 1 and otherwise equal zero if the neighbours of a node are not connected together. Average clustering coefficients (C_i) of 0.007, 0.016 and 0.026 were observed respectively for networks 1, 2 and 3 respectively. This further indicates that real world networks embedded in 2 dimensional Euclidean space cannot be random networks. The degrees of these networks vary between 1 and 5 with 5 being the maximum degree. This range was observed due to the constraint imposed by land use through population growth.

Degree and path lengths are significantly correlated i.e. lengthy streets tend to have more streets intersecting with them. The low clustering coefficient of the three graphs under study can be linked to planarity of most of the street networks.

During the network evolution, important properties such as clustering coefficients may likely change quite suddenly due to the growing probability of connection of some particular nodes. Table 2 shows the values of the average path lengths (L) and clustering coefficients (C) for these networks as compared with their random values L_{rand} and C_{rand} . Also, previous research has shown that the average path lengths and the average clustering coefficients of a random graph can be obtained using equations 3.3 and 3.4.

Table 2. Comparison between the average clustering coefficients (C_i) and average path lengths (L) of the networks and their corresponding values when assumed to be randomised.

Network	Average path length (L)	Average clustering coefficient (C_i)	L_{rand}	C_{rand}
1	0.066	0.007	0.313	0.020
2	0.088	0.016	0.347	0.023
3	0.062	0.026	0.283	0.023

$$L \sim \frac{\ln N}{\ln (\langle k \rangle - 1)} \quad (3.3)$$

$$C \sim \frac{\langle k \rangle}{N} \quad (3.4)$$

where L is the average path length and C is the average clustering coefficient.

However, the values of average clustering coefficients and path lengths obtained from the analysis carried out (Table 2) deviate from those obtained from the randomised version of the graphs using the formula stated in equations 3.3 and 3.4.

CONCLUSION

The degrees of the networks studied were observed to be correlated. This correlation can be linked to spatial constraints imposed on land based transportation by the land use. This confirms that transportation networks have spatial restrictions which have strong implications on the network topology as observed by the presence of degree correlation.

The three sections of urban street networks studied showed similar properties in terms of average clustering coefficients and path lengths. Small average clustering coefficients of 0.007, 0.016, and 0.026 for networks 1, 2 and 3 respectively suggest that the number of edges a node can connect in land based transportation is limited. Hence, it can be inferred that any one of these sections as well as its properties can be representative of Akure network as a whole.

Conditions imposed by planarity also have consequent effects on the ability of nodes in such networks to form giant clusters. Small clustering coefficients over the networks suggested that most of the nodes in land based transportation are limited to certain numbers of links. Also, small (average) path lengths of 0.066, 0.088 and 0.062 obtained for networks 1, 2 and 3 respectively suggest that nodes in the networks can be reached over short distances. The way the node degrees were distributed for each network seems to follow a power law which confirms Akure network as a scale free network.

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